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Noise-based logic: Binary, multi-valued, or fuzzy, with optional superposition of logic states

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ABSTRACT

A new type of deterministic (non-probabilistic) computer logic system inspired by the stochasticity of brain signals is shown. The distinct values are represented by independent stochastic processes: independent voltage (or current) noises. The orthogonality of these processes provides a natural way to construct binary or multi-valued logic circuitry with arbitrary number N of logic values by using analog circuitry. Moreover, the logic values on a single wire can be made a (weighted) superposition of the N distinct logic values. Fuzzy logic is also naturally represented by a two-component superposition within the binary case ($N = 2$). Error propagation and accumulation are suppressed. Other relevant advantages are reduced energy dissipation and leakage current problems, and robustness against circuit noise and background noises such as $1/f$, Johnson, shot and crosstalk noise. Variability problems are also non-existent because the logic value is an AC signal. A similar logic system can be built with orthogonal sinusoidal signals (different frequency or orthogonal phase) however that has an extra $1/N$ type slowdown compared to the noise-based logic system with increasing number of N furthermore it is less robust against time delay effects than the noise-based counterpart.

1. Introduction

Recently, new, non-conventional ways of stealth communications [1] and unconditionally secure communications [2] were introduced and the last one was successfully demonstrated [3]. These methods used electronic thermal noise and its enhanced versions as information carrier. Inspired by this success, a new type of computation was envisioned where the thermal noise and its statistical properties would be the information carrier [4]. For a short review of the results in [1–4] and that of other non-conventional informatics, see [5]. In [4] no concrete solution could be proposed for thermal noise-driven computers, however a lower limit of the energy dissipation in the order of kT/bit was given, where k is the Boltzmann constant and T is the temperature.

One reason to explore such an unconventional way of computing was the fact that neural signals are stochastic processes, thus the brain is also using noise and its statistical properties for information processing. Another set of reasons was the numerous problems with current microprocessors and the miniaturization to follow Moore's law [6]. Today's computer logic circuitry is a system of coupled DC amplifier stages and this situation represents enhanced vulnerability against *variability* [7] of fabrication parameters such as threshold voltage-inaccuracies in CMOS. Thin oxides imply great power dissipation due to leakage currents [8]. Thermal noise, its error generation and the power dissipation need of reducing these problems are another issue [9–11]. Famous initiatives such as quantum computers or reversible computing are unable to help in these regards [12,13]. Tough there are various interesting proposals [14–19] to "live with noise" and to improve these conditions, major breakthroughs are still needed to secure future evolution of performance.

As a next step forward in the direction shown in papers [1–4], in this Letter, we introduce a new, non-conventional logic initiative where the logic values are carried by independent noise processes. At the end of the Letter, we briefly show and compare another possibility, when the logic values are carried by sinusoidal signals with different frequencies or orthogonal initial phase, however the noise-based logic seems more feasible.

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2. Motives, and introducing noise-based logic

A general problem with today's binary logic is that, the non-zero DC voltage levels representing the basic logic values $(0, 1)$ ("Low", "High") can be considered vectors with different length but in the same direction. These vectors are not *orthogonal* to each other, or to background noises, transients/spikes, such as ground-plane EMI and cross-talk pulses because these all can be considered as parallel vectors. Thus, the shrinking noise margin with miniaturization implies a progressively increasing rate of dynamical errors.

In the *noise-based logic* scheme proposed in this Letter, the logic values are represented by *independent stochastic processes* (electronic noises) of *zero mean*. A noise-based logic gate typically contains analog circuitry, such as linear amplifiers, multipliers, filters (especially time averaging units) and analog switches. This situation has several immediate advantages:

(i) The different basic logic values are orthogonal not only to each other but also to any transients/spikes or any background noise including thermal noise or circuit noise, such as $1/f$, *shot*, *generation-recombination*, etc., processes.
(ii) Due to the zero mean of the stochastic processes, the logic values are AC signals and AC coupling can make it sure that the variability-related vulnerabilities are strongly reduced.
(iii) Switching errors. The noise-based logic values have a reduced impact because these switching errors are also independent stochastic processes thus also orthogonal to the logic values. This property implies a reduced energy need to run the switches. Moreover, these errors do not propagate and accumulate in the proposed noise-based logic system.
(iv) Due to the orthogonality and AC aspects (points (i) and (ii)), a noise carrying a given logic value on the data bus can have much smaller effective value than the power supply voltage of the chip. This property is also very different from today's digital circuitry and it can offer another way of reducing the energy consumption.

3. Mathematical definition of noise-based logic

Mathematically, noise-based logic is based on an orthogonal basis of time functions $V_i(t)$ $(i = 1, \ldots, N)$ similarly to Fourier analysis/synthesis, however here the base functions are independent electronic noises: different realizations of the same or different Gaussian stochastic processes with zero mean. Each base function represents a different logic value and N is the total number of logic base values. For example, in the binary case $(N = 2)$:

$$\langle L^2(t)\rangle = 1, \qquad \langle H^2(t)\rangle = 1 \quad \text{and} \quad \langle H(t)L(t)\rangle = 0, \tag{1}$$

where $\langle\,\rangle$ represents time average; the $L(t)$ and $H(t)$ noise processes represent the "Low" and "High" logic values, respectively; and for simplicity we supposed that the *RMS* value of the noises is 1. Note, that unlike in digital systems, the "Low" state has the same signal amplitude as the "High" state. Generally, for arbitrary number of logic values:

$$\langle V_i(t)V_j(t)\rangle = \delta_{i,j} \tag{2}$$

where $\delta_{i,j}$ is the Kronecker symbol (for $i = j$, $\delta_{i,j} = 1$, otherwise $\delta_{i,j} = 0$). Due to Eq. (1) or (2), the $L(t)$, $H(t)$ or $V_i(t)$ processes can be represented by orthogonal unit vectors in multidimensional space, thus we can use the term *logic base vectors* for *logic base values* and talk about N-dimensional logic space and *logic state vectors* in it.

Generally, a logic state vector is the *weighted* superposition of logic base vectors:

$$X(t) = \sum_{i=1}^{N} a_i V_i(t). \tag{3}$$

If only one of the a_i coefficients differs from zero, we have a "clean" multi-value logic otherwise we have a superposition which can be either a discrete superposition with discrete coefficient (for example $a_i = 0$ or $a_i = 1$), or continuum, fuzzy logic (for example $0 \leqslant a_i \leqslant 1$).

4. An example of coefficients: Binary noise-based logic scheme, and its fuzzy version

When $N = 2$, see above, and the coefficients a_1 and a_2 are either 0 or 1 and $a_1 \neq a_2$, we have a standard binary noise-based logic, see Fig. 1, were the vector representation of noise-based logic with the two base values is shown. When the logic is binary, only single base vectors are used. If a superposition $X(t) = a_L L(t) + a_H H(t)$ is used, it is a fuzzy logic with continuum values along the unit circle if we suppose that the normalization condition, $a_L^2 + a_H^2 = 1$, holds. Without normalization, the vector can point anywhere in that quarter plane $(x, y \geqslant 0)$.

5. General classes of noise-based logic

In general, the noise-based logic gates to be described below can work in the following modes, depending on their infrastructure. Note, superposition means the superposition of logic states on a single wire.

1. *Discrete state vectors* in finite, N-dimensional logic space, *without superposition*; see the $N = 2$ case in Fig. 1. Even though the system works with the noise-based logic values and with building elements of analog circuitry (see below), the practical accuracy of this system is identical to that of a *digital circuitry*. However in several practical aspects it is superior to the normal two-value (binary) logic, even when the noise based-logic is used in the binary mode, see points (i)–(iv) above.

2. *Discrete state vectors* in finite, N-dimensional logic space, *with superposition* (with discrete coefficients). Same as at point 1, however with superposition of the discrete logic values with *discrete, digital-accuracy coefficients*. In the binary case $(N = 2)$, such a superposition represents the usual fuzzy logic, see the superposition vector in Fig. 1. The simplest situation of the $N > 2$ case is when the coefficients a_i can only be 1 or 0 (a base element is "on" or "off" in the superposition). In the more general cases, to provide high resolution for