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Effect of the induced magnetic field on peristaltic flow of a couple stress fluid

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ABSTRACT

We have analyzed the MHD flow of a conducting couple stress fluid in a slit channel with rhythmically contracting walls. In this analysis we are taking into account the induced magnetic field. Analytical expressions for the stream function, the magnetic force function, the axial pressure gradient, the axial induced magnetic field and the distribution of the current density across the channel are obtained using long wavelength approximation. The results for the pressure rise, the frictional force per wave length, the axial induced magnetic field and distribution of the current density across the channel have been computed numerically and the results were studied for various values of the physical parameters of interest, such as the couple stress parameter γ, the Hartmann number M, the magnetic Reynolds number R_m and the time averaged mean flow rate θ. Contour plots for the stream and magnetic force functions are obtained and the trapping phenomena for the flow field is discussed.

1. Introduction

The study of a couple stress fluid is very useful in understanding various physical problems, because it possesses the mechanism to describe rheologically complex fluids such as liquid crystals, colloidal fluids, liquids containing long-chain molecules as polymeric suspensions, animal and human blood and lubrication.

The micro-continuum theory of couple stress fluid proposed by Stokes [1], defines the rotational field in terms of the velocity field for setting up the constitutive relationship between the stress and strain rate. Stokes micro-continuum theory is the simplest generalization of the classical theory of fluids, which allows for polar effects such as the presence of couple stresses, body couples and a non-symmetric stress tensor. Some theoretical studies [2–6] of blood flow indicate that some of the non-Newtonian flow properties of blood may be explained by assuming the blood to be a fluid with couple stress. The couple-stress fluid may be considered as a special case of a non-Newtonian fluid which is intended to take into account the particle size effects. Moreover, the couple stress fluid model is one of the numerous models that proposed to describe response characteristics of non-Newtonian fluids. The constitutive equations in these fluid models can be very complex and involving a number of parameters, also the outcoming flow equations lead to boundary value problems in which the order of differential equations is higher than the Navier–Stokes equations. Some recent investigations regarding such fluids are mentioned in the studies [7–15].

Magnetohydrodynamics (MHD) is the science which deals with the motion of a highly conducting fluids in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field, and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid [16]. The magnetohydrodynamic (MHD) flow of a fluid in a channel with elastic, rhythmically contracting walls (peristaltic flow) is of interest in connection with certain problems of the movement of conductive physiological fluids, e.g., the blood, blood pump machines and with the need for theoretical research on the operation of a peristaltic MHD compressor. Effect of a moving magnetic field on blood flow was studied by Stud et al. [17], and they observed that the effect of suitable moving magnetic field accelerates the speed of blood. Srivastava and Agrawal [18] considered the blood as an electrically conducting fluid and constitutes a suspension of red cell in plasma. Also Agrawal and Anwaruddin [19] studied the effect of magnetic field on blood flow by taking a simple mathematical model for blood through an equally branched channel with flexible walls executing peristaltic waves using long wavelength approximation method and observed, for the flow blood in arteries with arterial disease like arterial stenosis or arteriosclerosis, that the influence of magnetic field may be utilized as a blood pump in carrying out cardiac operations. Moreover, the principle of magnetic field in the form of a device called Magnetic Resonance Imaging (MRI). Now MRI is used for diagnosis of brain, vascular diseases and all the human body.

It has now been accepted that most of the physiological fluids behave like a non-Newtonian fluids. This approach provides a satisfactory understanding of the peristaltic mechanism involved in small blood vessels, lymphatic vessels, intestine, ductus efferentes

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of the male reproductive tract and in transport of spermatozoa in the cervical canal. Some recent studies [20–31] have considered the effect of a magnetic field on peristaltic flow of a Newtonian and non-Newtonian fluids, and in all of these studies the effect of the induced magnetic field have been neglected.

The first investigation of the effect of the induced magnetic field on peristaltic flow was studied by Vishnyakov and Pavlov [32] where they considers the peristaltic MHD flow of a conductive Newtonian fluid, they used the asymptotic narrow-band method to solve the problem and only obtained the velocity profiles in certain channel cross-sections for definite parameter values. Currently, there is only one attempt [33] for a non-Newtonian fluid (biviscosity fluid), where the authors analyzed the peristaltic flow of an incompressible electrically conducting biviscosity fluid through an axisymmetric non-uniform tube, under the considerations of long wavelength and low Reynolds number. They obtained the analytic expressions for the axial velocity and the axial induced magnetic field and they discussed the effect of the parameters involved in the problem on the pressure rise, frictional force per wavelength and the axial induced magnetic field without considering the magnetic force function, current density distribution and the trapping phenomena.

With the above discussion in mind, the goal of this investigation is to study the effect of the induced magnetic field on peristaltic flow of a couple stress fluid (as a blood model). The flow analysis is developed in a wave frame of reference moving with the velocity of the wave. This Letter runs as follows. In Section 2, the problem is first modeled and the non-dimensional governing equations are formulated in wave frame. The non-dimensional governing equations under the long wavelength and low Reynolds number approximation and the corresponding boundary conditions are prescribed in Section 3. Section 4 includes the exact solution of the problem. The results for the pressure rise, frictional force per wave length, the axial induced magnetic field and the distribution of the current density across the channel have been discussed for various values of the problem parameters in Section 5. Also, the pumping characteristics, the contour plot for the magnetic force function and the trapping phenomena are discussed in detail in the same section. Finally, the main conclusions are summarized in Section 6.

2. Mathematical model and the governing equations

Consider the unsteady hydro-magnetic flow of a viscous, incompressible and electrically conducting couple stress fluid through an axisymmetric two-dimensional channel of uniform thickness with a sinusoidal wave traveling down its wall. We choose a rectangular coordinate system for the channel with X' along the centerline in the direction of wave propagation and Y' transverse to it. The system is stressed by an external transverse uniform constant magnetic field of strength H'_0, which will give rise to an induced magnetic field $H'(h'_{X'}(X',Y',t'), h'_{Y'}(X',Y',t'), 0)$ and the total magnetic field will be $H'^{+}(h'_{X'}(X',Y',t'), H'_0 + h'_{Y'}(X',Y',t'), 0)$. The plates of the channel are assumed to be non-conductive and the geometry of the wall surface is defined as

$$h'(X',t') = a + b\sin\frac{2\pi}{\lambda}(X' - ct'), \tag{2.1}$$

where a is the half-width at the inlet, b is the wave amplitude, λ is the wavelength, c is the propagation velocity and t' is the time.

In the absence of the body couples, the governing equations for a magneto couple stress fluid are [1,5,33]:

(i) Maxwell's equations

$$\nabla\cdot\vec{H}' = 0, \qquad \nabla\cdot\vec{E}' = 0, \tag{2.2}$$

$$\nabla\wedge\vec{H}' = \vec{J}', \quad \text{with } \vec{J}' = \sigma\left\{\vec{E}' + \mu_e(\vec{V}'\wedge\vec{H}'^{+})\right\}, \tag{2.3}$$

$$\nabla\wedge\vec{E}' = -\mu_e\frac{\partial\vec{H}'}{\partial t'}. \tag{2.4}$$

(ii) The continuity equation

$$\nabla\cdot\vec{V}' = 0. \tag{2.5}$$

(iii) The Navier–Stokes equations

$$\begin{aligned}\rho\left\{\frac{\partial\vec{V}'}{\partial t'} + (\vec{V}'\cdot\nabla)\vec{V}'\right\} &= -\nabla\left(p' + \frac{1}{2}\mu_e(H'^{+})^2\right)\\ &\quad + \mu\nabla^2\vec{V}' - \eta\nabla^4\vec{V}' - \mu_e(\vec{H}'^{+}\cdot\nabla)\vec{H}'^{+},\end{aligned} \tag{2.6}$$

$$\nabla^2 = \frac{\partial^2}{\partial X'^2} + \frac{\partial^2}{\partial Y'^2}, \qquad \nabla^4 = \nabla^2\nabla^2,$$

where $\vec{V}'$ is the velocity vector, μ is the viscosity of the fluid, p' is the pressure, $\vec{E}'$ is an induced electric field, $\vec{J}'$ is the electric current density, μ_e is the magnetic permeability, η is a constant associated with the couple stress and σ is the electrical conductivity.

Combining Eqs. (2.2) and (2.3)–(2.5) we obtain the induction equation

$$\frac{\partial\vec{H}'^{+}}{\partial t'} = \nabla\wedge\{V'\wedge\vec{H}'^{+}\} + \frac{1}{\zeta}\nabla^2\vec{H}'^{+}, \tag{2.7}$$

where $\zeta = \frac{1}{\sigma\mu_e}$ is the magnetic diffusivity.

We shall carry out this investigation in a coordinate system moving with the wave speed c, in which the boundary shape is stationary. The coordinates and velocities in the laboratory frame (X', Y') and the wave frame (x', y') are related by:

$$\begin{aligned} x' &= X' - ct', \qquad y' = Y',\\ u' &= U' - c, \qquad v' = V',\end{aligned} \tag{2.8}$$

where U', V' and u', v' are the velocities components in the corresponding coordinate systems.

Using these transformation and introducing the following dimensionless variables

$$\begin{aligned} &x = \frac{x'}{\lambda}, \qquad y = \frac{y'}{a}, \qquad u = \frac{u'}{c}, \qquad v = \frac{v'}{c}, \qquad h = \frac{h'(x')}{a},\\ &p = \frac{a^2}{\lambda\mu c}p'(x'), \qquad t = \frac{ct'}{\lambda}, \qquad \psi = \psi'/ca, \qquad \phi = \frac{\phi'}{H_0 a},\end{aligned} \tag{2.9}$$

we find that, the equations which govern the MHD flow for a couple stress fluid in terms of the stream function $\psi(x, y)$ and magnetic-force function $\phi(x, y)$ are:

$$\begin{aligned} &Re\,\delta\left\{\left(\psi_y\frac{\partial}{\partial x} - \psi_x\frac{\partial}{\partial y}\right)\psi_y\right\}\\ &\quad = -\frac{\partial p_m}{\partial x} + \nabla^2\psi_y - \frac{1}{\gamma^2}\nabla^4\psi_y + Re\,S^2\phi_{yy}\\ &\qquad + Re\,S^2\delta\left(\phi_y\frac{\partial}{\partial x} - \phi_x\frac{\partial}{\partial y}\right)\phi_y,\end{aligned} \tag{2.10}$$

$$\begin{aligned} &Re\,\delta^3\left\{\left(\psi_x\frac{\partial}{\partial y} - \psi_y\frac{\partial}{\partial x}\right)\psi_x\right\}\\ &\quad = -\frac{\partial p_m}{\partial y} - \delta^2\nabla^2\psi_x + \frac{1}{\gamma^2}\nabla^4\psi_x - Re\,S^2\delta^2\phi_{xy}\\ &\qquad - Re\,S^2\delta^3\left(\phi_y\frac{\partial}{\partial x} - \phi_x\frac{\partial}{\partial y}\right)\phi_x,\end{aligned} \tag{2.11}$$

$$\psi_y - \delta(\psi_y\phi_x - \psi_x\phi_y) + \frac{1}{R_m}\nabla^2\phi = E, \tag{2.12}$$