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Exponential synchronization of complex networks with Markovian jump and mixed delays ☆

Yurong Liu [a,*], Zidong Wang [b], Xiaohui Liu [b]

[a] *Department of Mathematics, Yangzhou University, Yangzhou 225002, PR China*

[b] *Department of Information Systems and Computing, Brunel University, Uxbridge, Middlesex, UB8 3PH, United Kingdom*

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Abstract

In this Letter, we investigate the exponential synchronization problem for an array of N linearly coupled complex networks with Markovian jump and mixed time-delays. The complex network consists of m modes and the network switches from one mode to another according to a Markovian chain with known transition probability. The mixed time-delays are composed of discrete and distributed delays, both of which are mode-dependent. The nonlinearities imbedded with the complex networks are assumed to satisfy the sector condition that is more general than the commonly used Lipschitz condition. By making use of the Kronecker product and the stochastic analysis tool, we propose a novel Lyapunov–Krasovskii functional suitable for handling distributed delays and then show that the addressed synchronization problem is solvable if a set of linear matrix inequalities (LMIs) are feasible. Therefore, a unified LMI approach is developed to establish sufficient conditions for the coupled complex network to be globally exponentially synchronized in the mean square. Note that the LMIs can be easily solved by using the Matlab LMI toolbox and no tuning of parameters is required. A simulation example is provided to demonstrate the usefulness of the main results obtained.

Keywords: Synchronization; Complex network; Markovian Jumping; Discrete time-delay; Distributed time-delay; Kronecker product; Linear matrix inequality

1. Introduction

The last decade has witnessed rapidly growing research interests on the dynamics analysis of complex networks since the pioneering work of Watts and Strogatz [1]. The main reason is twofold: (1) complex network exists in our daily life with examples including the Internet, the World Wide Web (WWW), the World Trade Web, linguistic webs and food webs, etc.; (2) dynamical behaviors of complex networks have found numerous applications in various fields such as physics, technology, and the life sciences. Synchronization has proven to be one of the most important controlling activities to excite the collective behavior of complex dynamical networks, and has received increasing research attention in, for example, the large-scale and complex networks of chaotic oscillators [2–5], the coupled systems exhibiting spatio-temporal chaos and autowaves [6,7], and the array of coupled neural networks [8–10].

On the other hand, time delays often occur in complex networks because of the limited speed of signals travelling through the links [11–14] and the frequently delayed couplings in biological neural networks, gene regulatory networks, communication

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* Corresponding author.
E-mail address: liuyurong@gmail.com (Y. Liu).

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networks and electrical power grids [4,12,15]. It has been well known that time delays can cause complex dynamics such as periodic or quasi-periodic motions, Hopf bifurcation and higher-dimensional chaos. According to the way it arises, the time-delay can be generally categorized as two classes: discrete and distributed. Note that continuously distributed delays have gained particular attention since a network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths [16,17]. Recently, synchronization problems for various networks with discrete and/or distributed time-delays have extensively studied, see e.g. [4,10,12,15,18,19] and the references cited therein.

In reality, complex networks may exhibit a special characteristic called network mode switching. For example, in [20–22], it has been revealed that a neural network sometimes has finite modes that switch from one to another at different times, and such a switching (or jumping) can be governed by a Markovian chain. In [23], the bufferless packet *switching* of trees and leveled networks has been illustrated to be achievable with certain network topologies. In [24], a sensor network has been shown to have *jumping* behavior due to the network's working environment (normal or hazardous) and the mobility of sensor node. In [25], it has been concluded that the shuffle-exchange networks can model practical interconnection systems due to their size of its *switching* (*jumping*) elements and uncomplicated configuration. In [26], the associative memory of a stochastic Hopfield-like neural automata has been reported to *jump* between pattern attractors. It is worth mentioning that the control and filtering problems for dynamical systems with Markovian jumping parameters have already been widely studied, see e.g. [22,27–29] and the references therein. Although complex networks with Markovian jumping parameters have great application potential in a variety of areas, there has been very little existing literature on the synchronization problem for Markovian jumping complex networks with or without mixed time-delays, and the purpose of this Letter is therefore to shorten such a gap.

In this Letter, we deal with the synchronization problem for an array of coupled complex networks with simultaneous presence of both the discrete and distributed time-delays. The addressed complex network consists of m modes and the network switches from one mode to another according to a Markovian chain with known transition probability. By utilizing a novel Lyapunov–Krasovskii functional and the Kronecker product, we show that the addressed synchronization problem is solvable if a set of linear matrix inequalities (LMIs) are feasible. The main novelty of this Letter can be summarized as follows: (1) a new class of complex networks is proposed that contain Markovian jumping parameters; (2) both the discrete and distributed time-delays are considered that are dependent on the jumping mode; (3) rather than the commonly used Lipschitz-type function, a more general sector-like nonlinear function is employed; and (4) a new Lyapunov–Krasovskii functional is exploited to cater the mode-dependent distributed delays. A simulation example is provided to show the usefulness of the proposed global synchronization condition.

Notations. The notations are quite standard. Throughout this Letter, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript "T" denotes matrix transposition and the notation $X \geqslant Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X-Y$ is positive semidefinite (respectively, positive definite). Let I_n be the $n\times n$ identity matrix, and $|\cdot|$ denote the Euclidean norm in $\mathbb{R}^n$. If A is a square matrix, denote by $\lambda_{\max}(A)$ (respectively, $\lambda_{\min}(A)$) means the largest (respectively, smallest) eigenvalue of A. The notation $A\otimes B$ stands for the Kronecker product of matrices A and B. For $h>0$, $C([-h,0];\mathbb{R}^n)$ denotes the family of continuous functions φ from $[-h,0]$ to $\mathbb{R}^n$ with the norm $\|\varphi\| = \sup_{-h\leqslant\theta\leqslant 0}|\varphi(\theta)|$, $l_2[0,\infty]$ is the space of square integrable vector. Moreover, let $(\boldsymbol{\Omega},\mathcal{F},\{\mathcal{F}_t\}_{t\geqslant 0},\mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e., the filtration contains all P-null sets and is right continuous). Denote by $L^p_{\mathcal{F}_0}([-h,0];\mathbb{R}^n)$ the family of all $\mathcal{F}_0$-measurable $C([-h,0];\mathbb{R}^n)$-valued random variables $\xi=\{\xi(\theta)\colon -h\leqslant\theta\leqslant 0\}$ such that $\sup_{-h\leqslant\theta\leqslant 0}\mathbb{E}|\xi(\theta)|^p<\infty$ where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$. The asterisk $*$ in a symmetric matrix is used to denote term that is induced by symmetry. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

2. Problem formulation

Let $r(t)$ $(t\geqslant 0)$ be a right-continuous Markovian chain on a probability space $(\boldsymbol{\Omega},\mathcal{F},\{\mathcal{F}_t\}_{t\geqslant 0},\mathcal{P})$ taking values in a finite state space $\mathcal{N}=\{1,2,\dots,m\}$ with generator $\Pi=\{\pi_{ij}\}$ given by

$$P\big\{r(t+\Delta)=j \mid r(t)=i\big\}=\begin{cases}\pi_{ij}\Delta+o(\Delta), & \text{if } i\neq j,\\ 1+\pi_{ij}\Delta+o(\Delta), & \text{if } i=j.\end{cases}$$

Here $\Delta>0$, and $\pi_{ij}\geqslant 0$ is the transition rate from i to j if $j\neq i$ while

$$\pi_{ii}=-\sum_{j\neq i}\pi_{ij}.$$

Consider the following complex dynamical network coupled by N identical nodes with Markovian jumping parameters and mixed time-delays:

$$\frac{dx_k(t)}{dt}=f\big(x_k(t),r(t)\big)+g\big(x_k(t-\tau_{1,r(t)})\big)+\int_{t-\tau_{2,r(t)}}^{t}h\big(x_k(s)\big)\,ds+\sum_{l=1}^{N}w_{kl}\Gamma_{r(t)}x_l(t),\quad k=1,2,\dots,N, \tag{1}$$