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Multisoliton solutions to a nonisospectral (2 + 1)-dimensional breaking soliton equation

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Abstract

A nonisospectral $(2+1)$-dimensional breaking soliton equation ($(2+1)$DBSE) is derived, which corresponds to the spectral parameter λ satisfying $2\lambda\lambda_y - \lambda_t = \lambda^2$. The bilinear form for the nonisospectral $(2+1)$DBSE is obtained and the multisoliton solutions are worked out by means of the Hirota method and Wronskian technique, respectively. The nonisospectral $(2+1)$-dimensional breaking nonlinear Schrödinger equation and its multisoliton solutions are also presented by reduction.

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Keywords: Nonisospectral breaking soliton equation; Hirota method; Wronskian technique; Reduction

1. Introduction

Bilinear method including Hirota method and Wronskian technique is an efficient method for searching for soliton solutions of the nonlinear evolution equations. Hirota method was first proposed by Hirota in 1971 to obtain the N-soliton solutions of the KdV equation [1]. Soliton solutions can also be written in Wronskian form, which was first introduced by Satsuma [2] in 1979. Taking the advantage that special structure of a Wronskian contributes simple forms of its derivatives, Freeman and Nimmo [3] developed the Wronskian technique which admits direct verifications of solutions in Wronskian form to the bilinear equations. A determinant with double Wronskian structure is a generalization comparing with the standard Wronskian. Many soliton equations, such as the nonlinear Schrödinger equation [4,5], the 2-dimensional Toda lattice [6], the AKNS hierarchy [7] and some equations constrained from the KP hierarchy [8] admit solutions in double Wronskian form.

The breaking soliton equations are a kind of nonlinear evolution equations which can be used to describe the $(2+1)$-dimensional interaction of a Riemann wave propagating along the y-axis with a long-wave propagating along the x-axis. As for as the $(2+1)$DBSE associated with the AKNS hierarchy, which was proposed by O.I. Bogoyovlenskii [9,10]. In Ref. [11], many symmetries was constructed by infinitesimal "dressing" method. In Ref. [12], N-soliton solutions and double Wronskian solution were worked out by bilinear method.

In recent years, there has been much interest in study of the variable coefficient generalizations of complete integrable nonlinear evolution equations [13–17]. In the present Letter, we aim to investigate the soliton solutions of the nonisospectral $(2+1)$DBSE associated with the AKNS hierarchy. We first deduce the nonisospectral $(2+1)$DBSE which corresponds to the spectral parameter λ satisfying $2\lambda\lambda_y - \lambda_t = \lambda^2$. Then it is transformed into the bilinear form by which N-soliton solutions in Hirota's form and

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Wronskian form are obtained. Besides, the nonisospectral $(2+1)$-dimensional breaking nonlinear Schrödinger equation and its multisoliton solutions are also presented by reduction.

The Letter is organized as follows. In Section 2 the nonisospectral $(2+1)$DBSE is derived and its Lax pair is given. In Section 3, we solve the nonisospectral $(2+1)$DBSE by Hirota method. In Section 4 solution in double Wronskian form is proven. In Section 5 the nonisospectral $(2+1)$-dimensional breaking nonlinear Schrödinger equation and its N-soliton solutions are given by reduction.

2. Lax integrability of the nonisospectral (2 + 1)DBSE

Consider the following spectral problem

$$\psi_x = M\psi, \quad M = \begin{pmatrix} -\lambda & q \\ r & \lambda \end{pmatrix}, \quad \lambda_x = 0, \tag{2.1}$$

with the time evolution

$$\psi_t = 2y\lambda\psi_y + N\psi, \quad N = \begin{pmatrix} A & B \\ C & -A \end{pmatrix}. \tag{2.2}$$

The compatibility of (2.1) and (2.2) gives the zero-curvature equation

$$M_t - N_x + [M, N] - 2y\lambda M_y = 0. \tag{2.3}$$

From (2.3), we have

$$A = \partial_x^{-1}(r, q)\begin{pmatrix} -B \\ C \end{pmatrix} + (\lambda_t - 2\lambda\lambda_y)x + A_0, \tag{2.4a}$$

$$\begin{pmatrix} q \\ r \end{pmatrix}_t = L\begin{pmatrix} -B \\ C \end{pmatrix} - 2\lambda\begin{pmatrix} -B \\ C \end{pmatrix} + 2(2\lambda\lambda_y - \lambda_t)\sigma\begin{pmatrix} xq \\ xr \end{pmatrix} + 2y\lambda\begin{pmatrix} q_y \\ r_y \end{pmatrix} - 2A_0\sigma\begin{pmatrix} q \\ r \end{pmatrix}, \tag{2.4b}$$

where A_0 is a constant and

$$L = \begin{pmatrix} -\partial_x + 2q\partial_x^{-1}r & 2q\partial_x^{-1}q \\ -2r\partial_x^{-1}r & \partial_x - 2r\partial_x^{-1}q \end{pmatrix}, \qquad \sigma = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2.4c}$$

Expanding $(-B, C)^T$ as $(-B, C)^T = \sum_{j=1}^{n}(-b_j, c_j)^T\lambda^{n-j}$ and taking $n = 2$, $2\lambda\lambda_y - \lambda_t = \lambda^2$, $A_0 = 0$ in Eq. (2.4), we have

$$\begin{cases} q_t = -q_x + q\partial_x^{-1}qr - \frac{1}{2}x(q_{xx} - 2q^2r) + y[-q_{xy} + 2q\partial_x^{-1}(qr)_y], \\ r_t = r_x - r\partial_x^{-1}qr + \frac{1}{2}x(r_{xx} - 2qr^2) + y[r_{xy} - 2r\partial_x^{-1}(qr)_y]. \end{cases} \tag{2.5}$$

The corresponding A, B and C in N are

$$A = -x\lambda^2 + \frac{1}{2}xqr + \frac{1}{2}\partial_x^{-1}qr + y\partial_x^{-1}(qr)_y, \tag{2.6a}$$

$$B = xq\lambda - \frac{1}{2}xq_x - \frac{1}{2}q - yq_y, \tag{2.6b}$$

$$C = xr\lambda + \frac{1}{2}xr_x + \frac{1}{2}r + yr_y. \tag{2.6c}$$

3. Bilinear form and multisoliton solutions

Introducing the dependent variable transformation

$$q = \frac{g}{f}, \qquad r = -\frac{h}{f}, \tag{3.1}$$

(2.5) can be transformed into the bilinear form

$$\left(D_t + \frac{x}{2}D_x^2 + yD_xD_y\right)g \cdot f = -g_x f, \tag{3.2a}$$

$$\left(D_t - \frac{x}{2}D_x^2 - yD_xD_y\right)h \cdot f = h_x f, \tag{3.2b}$$

$$D_x^2 f \cdot f - 2gh = 0, \tag{3.2c}$$