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Modeling of Lamb wave propagation in plate with two-dimensional phononic crystal layer coated on uniform substrate using plane-wave-expansion method

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Abstract

We show that the conversional three-dimensional plane wave expansion method can be revised to investigate the lamb wave propagation in the plate with two-dimensional phononic crystal layer coated on uniform substrate. We find that an imaginary three-dimensional periodic system can be constructed by stacking the studied plates and vacuum layers alternately, and then the Fourier series expansion can be performed. The difference between our imaginary periodic system and the true three-dimensional one is that, in our system, the Bloch feature of the wave along the thickness direction is broken. Three different systems are investigated by the proposed method as examples. The principle and reliability of the method are also discussed.

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Keywords: Phononic crystal; Lamb wave; Plane-wave-expansion method

1. Introduction

The propagation of elastic/acoustic waves in periodic heterogeneous materials called phononic crystal (PC) has received much attentions in the last decade [1–5]. The existence of full band gaps, which have been demonstrated by a lot of theoretical [1–3] and experimental [4] works, should have many potential applications. The suggested possible utilities of PC would be wave barrier, filter, focus lens, waveguide and so on. In the previous studies, most of works are concentrated on the bulk waves propagation, only few papers are devote to the surface wave propagating on the surface of the half-infinite system [6–12] or Lamb waves [13–18] propagating in finite-thickness plate. In fact, surface acoustic wave (SAW) and Lamb wave device are widely used as detectors and sensors in practice. So it is worthwhile to give more studies on this subject.

To investigate the wave propagation in the two-dimensional thin PC plate, several numerical schemes based on the plane wave expansion (PWE) method have been developed. In Ref. [13], Sigalas and Economous suggested a numerical method based on the classical plate theory, by which the Lamb wave in thin PC plate is investigated. However, as it was pointed out in Ref. [14], this classical-plate-theory based method can only be used in a relatively low frequency region and for plate with small thickness. Another kind of method for finite PC system is the so-called full three-dimensional (3D) PWE method. The main idea of which can be found in Refs. [11,15,16,19]. In this kind of method, the PWE procedure is used to obtain the general wave solution in the infinite system firstly, and then the stress-free boundary condition on the surface is used, which lead to a stress-free-boundary-

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condition determinant. For such a determinant, a root searching procedure is usually required to find its zeroes. The shortcoming of the numerical root searching procedure is that the desirable roots usually cannot be selected out easily when the order of the determinant is relatively large [15]. Recently, Hsu and Wu gave in Ref. [14] an alternative scheme based on the Mindlin's plate theory. According to the Mindlin's plate theory, the wave field along the thickness direction of the plate can be expressed as a power series in real-space if the plate is uniform along this direction. By their way, the Lamb wave dispersive relationship in the PC plate can be obtained more precisely and efficiently. But this Mindlin's-plate-theory based method is also restricted within small thickness plate and low frequency region, and furthermore, it is not easy to be extended to deal with general Lamb wave in heterogenetic system, such as the one investigated in Refs. [15,17].

From the viewpoint of whether the boundary condition is explicitly used or not, we can separate the above mentioned numerical schemes into two different kinds. The first kind is the full 3D PWE method and so on. In which, the general solution of the wave field in the infinite system is obtained by the PWE procedure firstly, and the boundary conditions are then used explicitly to determine the particular solution. The second kind is the methods given in Refs. [13] or [14]. In which, a special form of the wave solution which can satisfy the boundary condition (such as the power series in the Mindlin's plate theory) is given firstly, and then an eigenvalue equation can be obtained by a PWE procedure. So the boundary condition is unnecessary to be used explicitly in this them. It is obvious that a stress-free-boundary-condition determinant is inevitable for the first kind, which lead a tremendous calculation. Thus, the second kind of numerical method should be more efficient.

In the present Letter, we show that the wave solution along the thickness direction of the plate can be expressed by the popular Fourier series expansion (FSE), we will see that the numerical method based on this scheme can be easily performed, and by which the dispersive relationship of the Lamb wave (or the general Lamb wave) in PC plate (or PC plate with substrate) can be efficiently investigated.

2. Theory

The considered system, which is stacked by a PC layer with thickness t_p and an uniform substrate layer with thickness t_s, is shown in Fig. 1(a). The systems studied in Refs. [14,15] can be looked as the special cases with $t_s = 0$. Note that the upper surface of PC layer and the lower surface of uniform layer are stress free, that is to say the propagating wave is confined strictly in the structure. We find that it is possible to extend the confined waves in this plate (as a function of z) into a periodic function along z direction by an analytic prolongation procedure, which means an infinite periodic structure can be obtained by this procedure. Keep this in mind, we construct a three-dimensional periodic system with a super cell shown in Fig. 1(b), in which two vacuum layers with thickness t_v are used. We will show below that the wave solution in the considered finite plate should be the same as the solution in the imaginary periodical system we constructed.

We know that the wave solutions in the elastic plate with stress-free boundary condition shown in Fig. 1(a) can be determined completely by the elastic equation

$$\rho\frac{\partial^2 u_i}{\partial t^2} = (c_{ijkl}u_{k,l})_{,j} \tag{1}$$

and the stress-free boundary conditions on two free surface

$$T_{3j} = c_{3jkl}u_{k,l} = 0. \tag{2}$$

So we can say that the solutions of this equation should be unchanged if we can prove each single plate in our imaginary periodical system obeys Eq. (2). To show this, we consider a two-layer system stacked by solid material A and B along z direction. The continuum condition on their interface can be written as

$$u_i^A = u_i^B, \tag{3}$$

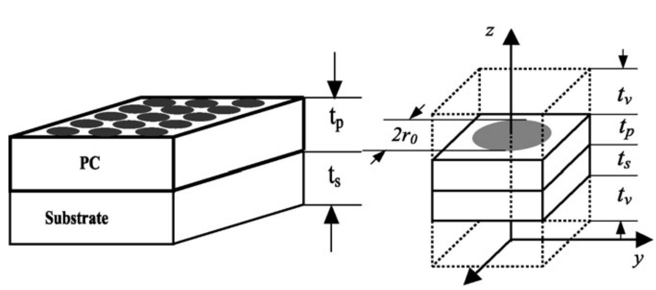

Fig. 1. (a) Square-lattice two-dimensional phononic crystal (PC) plate with substrate, t_p and t_s are the thickness of PC and substrate layer, respectively. (b) Unit cell of (a), two vacuum layers with thickness t_v are added on their outer surfaces. r_0 is the radius of the circular cylinder. The lattice constant in xy plane is a.