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Physics Letters A 362 (2007) 246–251

PHYSICS LETTERS A

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Possible localized transport induced by a wave propagating along a vacuum–matter interface

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Received 16 March 2006; accepted 9 October 2006

Available online 18 October 2006

Communicated by F. Porcelli

Abstract

Macroscopic derivation of the entrainment of matter induced by a surface elastic wave propagating along the flexible vacuum–matter interface is conducted by considering the nonlinear coupling between the interface and the rarefaction effect. The critical reflux values associated with the product of the second-order (unit) body forcing and the Reynolds number (representing the viscous dissipations) decrease as the Knudsen number (representing the rarefaction measure) increases from zero to 0.1. We obtained the matter-freezed or zero-volume-flow-rate states for specific Reynolds numbers and wave numbers which might be linked to the evolution of the Universe.

PACS: 02.30.Mv; 04.20.Ha; 98.80.Bp

Keywords: Casimir effect

1. Introduction

It is now widely accepted that the dominant cause of structure in the Universe is a spatial perturbation. This perturbation is present on cosmological scales a few Hubble times before these scales enter the horizon, at which stage it is time-independent with an almost flat spectrum. One of the main objectives of theoretical cosmology is to understand its origin [1]. The existence of an exotic cosmic fluid with negative pressure, which constitutes about the 70 percent of the total energy of the universe, has been perhaps the most surprising discovery made in cosmology. This dark energy is supported by the astrophysical data obtained from Wilkinson Microwave Anisotropy Probe (WMAP) (Map) and high redshift surveys of supernovae. The dark energy is considered a fluid characterized by a negative pressure and usually represented by the equation of state $\omega = p/\rho$, where ω lies very close to -1, most probably being below -1. The role of the dissipative processes in the evolution of the early universe has also been extensively studied. In the case of isotropic and homogeneous cosmologies, any dissipation process in a FRW cosmology is scalar, and therefore may be modeled as a bulk viscosity within a thermodynamical approach. A well-known result of the FRW cosmological solutions, corresponding to universes filled with perfect fluid and bulk viscous stresses, is the possibility of violating dominant energy condition (DEC), since $\rho + p < 0$ [3].

Recent evidence—microwave background anisotropies, complemented by data on distant supernovae—reveals that our Universe actually is ‘flat’, but that its dominant ingredient (ca. 70% of the total mass energy) is something quite unexpected: ‘dark energy’ pervading all space, with negative pressure. We do know that this material is very dark and that it dominates the internal kinematics, clustering properties and motions of galactic systems. Dark matter is commonly associated to weakly interacting particles (WIMPs), and can be described as a fluid with vanishing pressure. It plays a crucial role in the formation and evolution of structure in the Universe and it is unlikely that galaxies could have formed without its presence [1,2]. Analy-

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2006.10.028

sis of cosmological mixed dark matter models in spatially flat Friedmann Universe with zero Λ term have been presented before. The set of equations governing the evolution of the universe is completed by the Friedmann equations for the scale factor $(a(t))$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho + \Lambda) - \frac{k}{a^2},$$
$$\left(\frac{\ddot{a}}{a}\right)^2 = -\frac{4\pi G}{3}\big(\rho + \Lambda + 3(p - \Lambda)\big),$$

where $k = 0$ is for the flat Universe, G is the Newton's constant, ρ and p are the density and pressure [4]. A large majority of dark energy models describe dark energy in terms of the equation of state (EOS) $p_d = \omega\rho_d$, where ω is the parameter of the EOS, while p_d and ρ_d denote the pressure and the energy density of dark energy, respectively. The value $\omega = -1$ is characteristic of the cosmological constant, while the dynamical models of dark energy generally have $\omega \geqslant -1$.

It is convenient to express the mean densities ρ_i of various quantities in the Universe in terms of their fractions relative to the critical density: $\Omega_i = \rho_i/\rho_{\text{crit}}$. The theory of cosmological inflation strongly suggests that the total density should be very close to the critical one ($\Omega_{\text{tot}} \sim 1$), and this is supported by the available data on the cosmic microwave background (CMB) radiation. The fluctuations observed in the CMB at a level ca. 10^{-5} in amplitude exhibit a peak at a partial wave $l \sim 200$, as would be produced by acoustic oscillations in a flat Universe with $\Omega_{\text{tot}} \sim 1$. At lower partial waves, $l \gg 200$, the CMB fluctuations are believed to be dominated by the Sachs–Wolfe effect due to the gravitational potential, and more acoustic oscillations are expected at $l > 200$, whose relative heights depend on the baryon density Ω_b. At even larger values, $l \geqslant 1000$, these oscillations should be progressively damped away [4].

Influential only over the largest of scales—the cosmological horizon—is the outermost species of invisible matter: the vacuum energy [5] (also known by such names as dark energy, quintessence, x-matter, the zero-point field, and the cosmological constant Λ). If there is no exchange of energy between vacuum and matter components, the requirement of general covariance implies the time dependence of the gravitational constant G. Thus, it is interesting to look at the interacting behavior between the vacuum (energy) and the matter from the macroscopic point of view. One related issue, say, is about the dissipative matter of the flat Universe immersed in vacua [3] and the other one is the macroscopic Casimir effect with the deformed boundaries [6].

Theoretical and experimental studies of interphase nonlocal transport phenomena which appear as a result of a different type of nonequilibrium representing propagation of a surface elastic wave have been performed since late 1980s [7,8]. These are relevant to rarefied gases flowing along deformable elastic slabs with the dominated parameter being the Knudsen number (K_n = mean-free-path$/L_d$, mean-free-path (mfp) is the mean free path of the (matter) gas, L_d is proportional to the distance between two slabs) [9]. There will be a nonzero slip velocity (proportional to K_n) along the gas–solid interface due to incomplete accommodation of momentum between the colliding and reflecting particles [9,10].

The role of the Knudsen number is similar to that of the Navier slip parameter N_s [12]; here, $N_s = \mu S/d$ is the dimensionless Navier slip parameter; S is a proportionality constant as $u_s = S\tau$, τ: the shear stress of the bulk velocity; u_s: the dimensional slip velocity; for a no-slip case, $S = 0$, but for a no-stress condition: $S = \infty$, μ is the fluid viscosity, d is one half of the distance between upper and lower slabs.

A large-scale smoothed-out model of the universe ignores small-scale inhomogeneities, but the averaged effects of those inhomogeneities may alter both observational and dynamical relations at the larger scale. Here, the transport driven by the wavy elastic vacuum–matter interface will be presented. We adopt the macroscopic or hydrodynamical approach and simplify the original system of equations (related to the momentum and mass transport) to one single higher-order quasi-linear partial differential equation in terms of the unknown stream function. In this study, we shall assume that the Mach number $Ma \ll 1$, and the governing equations are the incompressible Navier–Stokes equations [12] which are associated with the relaxed slip velocity boundary conditions along the interfaces [9, 10]. We then introduce the perturbation technique so that we can solve the related boundary value problem approximately. To consider the originally quiescent gas for simplicity, due to the difficulty in solving a fourth-order quasi-linear complex ordinary differential equation (when the wavy boundary condition are imposed), we can finally get an analytically perturbed solution and calculate those physical quantities we have interests, like, time-averaged transport or entrainment, perturbed velocity functions, critical unit body forcing corresponding to the freezed or zero-volume-flow-rate states. These results might be closely linked to the vacuum–matter interactions and the evolution of the Universe.

2. Formulations

We consider a two-dimensional matter-region of uniform thickness which is approximated by a homogeneous rarefied gas (Newtonian viscous fluid). The equation of motion is

$$(\lambda + \mu)\,\text{grad}\,\text{div}\,\mathbf{u} + \mu\nabla\mathbf{u} + \rho\mathbf{p} = \rho\frac{\partial^2\mathbf{u}}{\partial t^2}, \tag{1}$$

where λ and μ are Lamé constants, $\mathbf{u}$ is the displacement field (vector), ρ is the mass density and $\mathbf{p}$ is the body force for unit mass. The Navier–Stokes equations, valid for Newtonian fluids (both gases and liquids), has been a mixture of continuum fluid mechanics ever since 1845 following the seemingly definitive work of Stokes and others [12], who proposed the following rheological constitutive expression for the fluid deviatoric or viscous stress (tensor) $\mathbf{T}$: $\mathbf{T} = 2\mu\nabla\mathbf{u} + \lambda\mathbf{I}\nabla\cdot\mathbf{u}$.

The flat-plane boundaries of this matter-region or the vacuum–matter interfaces are rather flexible and presumed to be elastic, on which are imposed traveling sinusoidal waves of small amplitude a (possibly due to vacuum fluctuations). The vertical displacements of the upper and lower interfaces ($y = L_d$ and $-L_d$) are thus presumed to be η and $-\eta$, re-