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# One plus two-body random matrix ensemble with spin: Analysis using spectral variances

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**Abstract**

For one plus two-body random matrix ensembles with spin, the chaos measure inverse participation ratio is calculated for 6, 7 and 8 fermion systems with good total spin $S$. Propagation equations for fixed-spin spectral variances explain the numerical results. Similarly, lower order correlations between spectra with different particle numbers and spins are analyzed using fixed-spin energy centroids and spectral variances. The correlations are found to be small for low-spin members.


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## 1. Introduction

Embedded Gaussian orthogonal ensembles of one plus two-body interactions [EGOE(1 + 2)] operating in many particle spaces [1], apply in a generic way to finite interacting fermion systems such as nuclei [1–3], atoms [4,5], quantum dots [6,7], small metallic grains [8], interacting spin systems modeling quantum computing core [9] etc. as the Hamiltonian ($H$) for these systems consists of a mean-field one-body [$h(1)$] plus a complexity generating two-body [$V(2)$] interaction. In the past several years EGOE(1 + 2) for spinless fermion systems has been studied in detail and it is established that, as the strength of the interaction changes, in terms of state density, level fluctuations, strength functions and entropy, the ensemble admits three chaos markers as summarized in [10]. These markers correspond to Poisson to GOE transition in level fluctuations [11], Breit–Wigner to Gaussian transition in strength functions [12,13] and the thermodynamic region [14]. In addition, the state density in general takes Gaussian form independent of the strength of the interaction [15]. These EGOE(1 + 2) results for spinless fermion systems apply in many situations to realistic systems such as nuclei, atoms, quantum dots etc. though they carry additional good quantum numbers [1,16]. For a better random matrix model, clearly EGOE with good quantum numbers (in addition to particle number $m$), i.e., EGOE with group symmetries, should be studied [17].

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Simple but non-trivial and useful extended EGOE ensemble for fermions is EGOE(1 + 2) with spin (**s**) degree of freedom. Hereafter this ensemble is called EGOE(1 + 2)-**s** and just EGOE(1 + 2) refers to spinless fermion systems. Let us consider $m$ fermions with each fermion carrying spin $\mathbf{s} = \frac{1}{2}$ degree of freedom and occupying $\Omega$ number of orbits so that the number of single particle states $N = 2\Omega$. For the $m$ fermion system with total spin $S$ a good quantum number, the EGOE(1 + 2)-**s** in $(m, S)$ spaces is generated by $U(N) \supset U(\Omega) \otimes SU(2)$ algebra with $SU(2)$ generating spin $S$. This ensemble has been studied numerically in the past for the ground state properties such as ground state spin structure in quantum dots and nuclei [7,18] and odd–even staggering in binding energies in small metallic grains [8]. The results here are explained in [7,8,17,18] by using approximate expressions for fixed spin spectral variances. Our purpose in this Letter and others to follow is to systematically study the properties of EGOE(1 + 2)-**s** ensemble and later extend it to EGOE(1 + 2)-$J$ ($J$ being total angular momentum) which is appropriate for nuclei and atoms [1–3,5]. Now we will give a preview.

In Section 2 the ensemble EGOE(1 + 2)-**s** is defined and a method for constructing this ensemble, for machine calculations, is given. Results for density of states and ground state spin structure are used as a test. Section 3 gives propagation equations for fixed spin energy centroids and variances for one plus two-body Hamiltonians that preserve $m$ particle spin $S$. For EGOE(1 + 2)-**s**, the chaos measure inverse participation ratio and lower order correlations between spectra with different particle numbers and spins are studied and the results are given in Sections 4 and 5 respectively. These results are explained using the propagation formulas for energy centroids and spectral variances. Finally Section 6 gives conclusions and future outlook.

## 2. EGOE(1 + 2)-s ensemble: definition and construction

Let us begin with a system of $m$ ($m > 2$) fermions distributed say in $\Omega$ number of single particle orbits each with spin $\mathbf{s} = \frac{1}{2}$ so that the number of single particle states $N = 2\Omega$. Single particle states are denoted by $|i, m_{\mathbf{s}} = \pm\frac{1}{2}\rangle$ with $i = 1, 2, \ldots, \Omega$ and similarly two particle antisymmetric stats are denoted by $|(ij)s, m_s\rangle_a$ with $s = 0$ or 1. For one plus two-body Hamiltonians preserving $m$ particle spin $S$, the one-body Hamiltonian is $h(1) = \sum_{i=1,2,\ldots,\Omega} \epsilon_i n_i$ where the orbits $i$ are doubly degenerate, $n_i$ are number operators and $\epsilon_i$ are single particle energies (it is in principle possible to consider $h(1)$ with off-diagonal energies $\epsilon_{ij}$). Similarly the two-body Hamiltonian $V(2)$ is defined by the two-body matrix elements $V^s_{ijkl} = {}_a\langle (kl)s, m_s | V(2) | (ij)s, m_s\rangle_a$ with the two-particle spin $s = 0, 1$ and they are independent of the $m_s$ quantum number; note that for $s = 1$, only $i \neq j$ and $k \neq l$ matrix elements exist. Thus $V(2) = V^{s=0}(2) + V^{s=1}(2)$; the sum here is a direct sum. Now, EGOE(2)-**s** for a given $(m, S)$ system is generated by defining the two parts of the two-body Hamiltonian to be independent GOEs (one for $V^{s=0}(2)$ and other for $V^{s=1}(2)$) in the 2-particle spaces and then propagating the $V(2)$ ensemble $\{V(2)\} = \{V^{s=0}(2)\} + \{V^{s=1}(2)\}$ to the $m$-particle spaces with a given spin $S$ by using the geometry (direct product structure) of the $m$-particle spaces; here { } denotes ensemble. Then EGOE(1 + 2)-**s** is defined by

$$\{H\}_{\text{EGOE(1+2)-}\mathbf{s}} = h(1) + \lambda_0\{V^{s=0}(2)\} + \lambda_1\{V^{s=1}(2)\}, \tag{1}$$

where $\{V--\}$ are GOEs with unit variance and $\lambda_0$ and $\lambda_1$ are the strengths of the $s = 0$ and $s = 1$ parts of $V(2)$, respectively. The mean-field one-body Hamiltonian $h(1)$ in Eq. (1) is a fixed one-body operator defined by the single particle energies $\epsilon_i$ with average spacing $\Delta$ (it is possible to draw the $\epsilon_i$'s from the eigenvalues of a random ensemble [7]). Thus, EGOE(1 + 2)-**s** in Eq. (1) is defined by the six parameters $(\Omega, m, S, \Delta, \lambda_0, \lambda_1)$ and without loss of generality we put $\Delta = 1$ so that $\lambda_0$ an $\lambda_1$ are in units of $\Delta$. Before proceeding further, it is useful to note that the $H$ matrix dimension $d(m, S)$ for a given $(m, S)$, i.e., number of levels in the $(m, S)$ space (with each of them being $(2S + 1)$-fold degenerate), is

$$d(m, S) = \frac{(2S+1)}{(\Omega+1)} \binom{\Omega+1}{m/2+S+1} \binom{\Omega+1}{m/2-S}. \tag{2}$$

They satisfy the sum rule $\sum_S (2S + 1) d(m, S) = \binom{N}{m}$. For example for $m = \Omega = 8$, the dimensions are 1764, 2352, 720, 63 and 1 for $S = 0$, 1, 2, 3 and 4, respectively.

In order to construct the many particle Hamiltonian matrix for a given $(m, S)$, first we consider the single particle states $|i, m_{\mathbf{s}} = \pm\frac{1}{2}\rangle$ and arrange them in such a way that the first $\Omega$ states have $m_{\mathbf{s}} = \frac{1}{2}$ and the remaining $\Omega$ states have $m_{\mathbf{s}} = -\frac{1}{2}$ so that a state $|r\rangle = |i = r, m_{\mathbf{s}} = \frac{1}{2}\rangle$ for $r \leqslant \Omega$ and $|r\rangle = |i = r - \Omega, m_{\mathbf{s}} = -\frac{1}{2}\rangle$ for $r > \Omega$. Now the $m$-particle configurations $\mathbf{m}$ and the corresponding $m_S$ values are

$$\mathbf{m} = (m_1, m_2, \ldots, m_\Omega, m_{\Omega+1}, m_{\Omega+2}, \ldots, m_{2\Omega}), \qquad m_r = 0 \text{ or } 1, \qquad m_S = \frac{1}{2}\left[\sum_{r=1}^{\Omega} m_r - \sum_{r'=\Omega+1}^{2\Omega} m_{r'}\right]. \tag{3}$$

It is important to note that the $\mathbf{m}$'s with $m_S = 0$ will contain states with all $S$ values for even $m$ and similarly with $m_S = \frac{1}{2}$ for odd $m$. Therefore we construct the $m$ particle Hamiltonian matrix using the basis defined by $\mathbf{m}$'s with $m_S = 0$ for even $m$ and $m_S = \frac{1}{2}$ for odd $m$. The dimension of this basis space is $\sum_S d(m, S)$. To proceed further, the (1 + 2)-body Hamiltonian defined by $(\epsilon_i, V^{s=0,1}_{ijkl})$'s should be converted into the $|i, m_{\mathbf{s}} = \pm\frac{1}{2}\rangle$ basis. Then $\epsilon_i$ change to $\epsilon_r$ with the index $r$ defined as above and $V^{s=0,1}_{ijkl}$