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New hyperbolic schemes for reliable treatment of Boussinesq equation

Abdul-Majid Wazwaz

Department of Mathematics and Computer Science, Saint Xavier University, Chicago, IL 60655, USA

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Abstract

In this work, the nonlinear Boussinesq equation is investigated. New schemes that employ hyperbolic functions are introduced to carry out this work. New solitary wave solutions and plane periodic solutions are established.
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1. Introduction

Nonlinear phenomena appear in a wide variety of scientific applications such as plasma physics, solid state physics, fluid dynamics and chemical kinetics [1–8]. Because of the increased interest in the theory of solitary waves, a broad range of analytical and numerical methods were used in the analysis of these scientific models.

The fourth-order nonlinear Boussinesq equation reads

$$u_{tt} - u_{xx} - 3\left(u^2\right)_{xx} - u_{xxxx} = 0. \tag{1}$$

Eq. (1) was introduced by Boussinesq to describe the propagation of long waves in shallow water. It also arises in other physical applications such as nonlinear lattice waves, iron sound waves in a plasma, and in vibrations in a nonlinear string. Moreover, it was applied to problems in the percolation of water in porous subsurface strata.

Travelling wave solutions appear in many forms. *Solitons*, which form the main type of travelling wave solutions, are localized travelling waves with infinite support asymptotically zero at large distances. Solitons appear in other forms such as *kink waves*, *peakons*, *cuspons*, and *compactons*. The latter are solitons with compact support where each compacton is a soliton confined to a finite core without exponential wings. Most recently, Rosenau and Pikovsky [6] introduced a new type of travelling wave, termed *kovaton*, which is a robust compacton with a flat top made of two compact kinks glued together. The discrete kovatons are hat-shaped, and kovatons collide elastically as compactons.

The appearance of these solitary wave solutions has increased the menagerie of solutions appearing in model equations, both integrable and nonintegrable [5,6]. All nonlinear equations that give rise to nonanalytic solutions have nonlinear dispersion as a distinguished feature.

It is the objective of this work to further complement previous studies to make a further progress in this field. We aim in this work to formally derive new sets of travelling wave solutions. Many strategies will be pursued to achieve our goal. The work rests mainly on new ansätze that use one hyperbolic function or combine two hyperbolic functions. The tanh method [7, 8] and the sine–cosine method [9–14] will be used as well.

In what follows, we highlight the main features of the proposed methods. The power of the methods, that will be used, is its ease of use to determine shock or solitary type of solutions.

2. The methods

In this section we present three ansätze, the cosh or sinh ansätze I and II, the sinh–cosh ansatz III, the tanh method and the sine–cosine method to handle the Boussinesq equation (1).

E-mail address: wazwaz@sxu.edu (A.-M. Wazwaz).

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The proposed ansätze I, II and III can be used directly in a straightforward manner to determine the unknown parameters involved in each ansatz.

2.1. A cosh or sinh ansatz I

In this case we introduce a cosh or a sinh ansatz I of the form

$$u(x,t) = \frac{\lambda}{1+\alpha\cosh[\mu(x-ct)]}, \tag{2}$$

or

$$u(x,t) = \frac{\lambda}{1+\alpha\sinh[\mu(x-ct)]}, \tag{3}$$

where λ, μ, and α are parameters that will be determined. This ansatz can be applied directly to the given equation to determine the parameters.

2.2. A cosh or sinh ansatz II

In this case we introduce a cosh or a sinh ansatz I of the form

$$u(x,t) = \frac{\lambda+\beta\cosh[\mu(x-ct)]}{1+\alpha\cosh[\mu(x-ct)]}, \tag{4}$$

or

$$u(x,t) = \frac{\lambda+\beta\sinh[\mu(x-ct)]}{1+\alpha\sinh[\mu(x-ct)]}, \tag{5}$$

where λ, μ, α and β are parameters that will be determined. This ansatz can also be applied directly to the given equation to determine the parameters.

2.3. A sinh–cosh ansatz III

In this case we introduce a sinh–cosh ansatz III of the form

$$u(x,t) = \frac{\lambda+\alpha\cosh[\mu(x-ct)]}{\sinh^2[\mu(x-ct)]}, \tag{6}$$

or

$$u(x,t) = \frac{\lambda+\alpha\sinh[\mu(x-ct)]}{\cosh^2[\mu(x-ct)]}, \tag{7}$$

where λ, μ, α and β are parameters.

2.4. The tanh method

The tanh method is a powerful solution method for the computation of exact travelling wave solutions. A power series in tanh was used to obtain analytical solutions of travelling wave type of certain nonlinear evolution equations.

The PDE in two independent variables

$$P(u, u_t, u_x, u_{xx}, u_{xxx}, \ldots) = 0, \tag{8}$$

can be transformed to a nonlinear ODE

$$Q(u, u', u'', u''', \ldots) = 0, \tag{9}$$

upon using a wave variable $\xi = x - ct$. Eq. (9) is then integrated as long as all terms contain derivatives where integration constants are considered zeros.

To avoid complexity, Malfliet and Hereman [7,8] had customized the tanh technique by introducing tanh as a new variable, since all derivatives of a tanh are represented by a tanh itself. Introducing a new independent variable

$$Y = \tanh(\mu\xi), \tag{10}$$

leads to the change of derivatives:

$$\frac{d}{d\xi} = \mu\left(1-Y^2\right)\frac{d}{dY},$$
$$\frac{d^2}{d\xi^2} = \mu^2\left(1-Y^2\right)\left(-2Y\frac{d}{dY} + \left(1-Y^2\right)\frac{d^2}{dY^2}\right), \tag{11}$$

where other derivatives can be derived in a similar manner. The following series expansion

$$u(\mu\xi) = S(Y) = \sum_{k=0}^{M} a_k Y^k, \tag{12}$$

is proposed, where M is a positive integer, in most cases, that will be determined. Substituting (11) and (12) into the simplified ODE yields an equation in powers of Y. The parameter M is usually obtained by balancing the linear terms of highest order in the resulting equation with the highest-order nonlinear terms. With M determined, we collect all coefficients of powers of Y in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters a_k, $(k = 0, \ldots, M)$, μ, and c. Having determined these parameters, and using (12) we obtain an analytic solution $u(x,t)$ in a closed form.

2.5. The sine–cosine method

The solutions of many nonlinear equations can be expressed in the form

$$u(x,t) = \begin{cases} \{\lambda\cos^\beta(\mu\xi)\}, & |\xi| \leqslant \frac{\pi}{2\mu}, \\ 0 & \text{otherwise}, \end{cases} \tag{13}$$

or in the form

$$u(x,t) = \begin{cases} \{\lambda\sin^\beta(\mu\xi)\}, & |\xi| \leqslant \frac{\pi}{\mu}, \\ 0 & \text{otherwise}, \end{cases} \tag{14}$$

where λ, μ, and β are parameters that will be determined, μ and c are the wave number and the wave speed, respectively. We then use

$$u(\xi) = \lambda\cos^\beta(\mu\xi),$$
$$\left(u^n\right)'' = -n^2\mu^2\beta^2\lambda^n\cos^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\cos^{n\beta-2}(\mu\xi), \tag{15}$$

and for (14) we use

$$u(\xi) = \lambda\sin^\beta(\mu\xi),$$
$$\left(u^n\right)'' = -n^2\mu^2\beta^2\lambda^n\sin^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\sin^{n\beta-2}(\mu\xi). \tag{16}$$

Substituting (15) or (16) into the reduced ODE gives a trigonometric equation of $\cos^R(\mu\xi)$ or $\sin^R(\mu\xi)$ terms. The parameters are then determined by first balancing the exponents of each

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