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Discrete soliton collisions in a waveguide array with saturable nonlinearity

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Abstract

We study the symmetric collisions of two mobile breathers/solitons in a model for coupled wave guides with a saturable nonlinearity. The saturability allows the existence of mobile breathers with high power. Three main regimes are observed: breather fusion, breather reflection and breather creation. The last regime seems to be exclusive of systems with a saturable nonlinearity, and has been previously observed in continuous models. In some cases a "symmetry breaking" can be observed, which we show to be an numerical artifact.

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1. Introduction

Since the 1960s, a great number of papers have considered the properties of solitons in nonlinear optic media with a Kerr-type (cubic) nonlinearity. This media can be modelled by the cubic nonlinear Schrödinger (NLS) equation. As it is well known, the NLS equation is integrable and, in consequence, solitons interact elastically [1].

More recently, several authors have studied the properties of solitons in photo-refractive media [2]. In this case, the equation describing these media is a modification of the original NLS, which consists in substituting the Kerr nonlinearity term by another one of saturable type. This saturable (SNLS) equation is nonintegrable and the soliton collision processes are inelastic, leading to annihilation, fusion or creation of solitons [3]. This last phenomena consists of the appearance of three solitons after the collision of only two of them. Another important feature of the SNLS is that the behaviour of the solutions is quite generic, being independent of the details of the mathematical model.

The *discrete* version of the NLS equation can be used to describe nonlinear waveguide arrays within the tight binding approximation [4]. The existence and properties of mobile discrete breathers/solitons in DNLS lattices has been considered in a number of studies. (We use the terms breathers and solitons interchangeably in this context, also intrinsic localized modes.) An early brief study [5] showed that breathers could propagate along the lattice with a small loss of energy, and could become trapped by inhomogeneities in the lattice. Later, a more detailed study [6] suggested that "exact" travelling breathers might exist, at least for some parameter ranges. The reviews [7,8] refer to many other papers in this area. More recently, work has concentrated on breathers with infinite oscillating tails [9], although the question of the *existence* of *exact* breather solutions which tend to zero as $n \to \pm\infty$ has not yet been resolved. Given the long history of mobile breather solutions of this equation, it is rather surprising that a systematic study of the collision of two breathers in the DNLS model has only recently been carried out [10]. (We mention also that collisions have been studied in generalised nearly integrable DNLS model in [11,12].)

Recently, some studies have considered the existence of mobile breathers in waveguide arrays in photo-refractive crys-

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tal, described by a DNLS equation with saturable nonlinearity [13,14]. In particular, these papers considered a discrete version of the Vinetskii–Kukhtarev model [2,15]. This model system, which we consider in this Letter, is governed by the following equation of motion

$$i\dot{u}_n - \beta\frac{u_n}{1+|u_n|^2} + (u_{n+1} - 2u_n + u_{n-1}) = 0. \qquad (1)$$

The key difference between the cubic DNLS equation and the saturable DNLS equation is that in the later, the Peierls–Nabarro barrier (the energy difference between a bond-centred and a site-centred breather with the same power) is bounded and, in most cases, smaller than in the former [16]. It allows the existence of mobile breathers of high power.

It is worth noting that there is another saturable DNLS equation in the literature, namely

$$i\dot{\psi}_n + \frac{\nu|\psi_n|^2}{1+\mu|\psi_n|^2}\psi_n + (\psi_{n+1} - 2\psi_n + \psi_{n-1}) = 0. \qquad (2)$$

For example, Khare et al. [17] have recently published an exact stationary breather solution for (2), although in fact the stationary solution of this equation is just the solution to an integrable map first published by McMillan in 1971 [18]. Maluckov et al. [19] have also recently studied stationary solutions of (2). However, the two models are not independent, solutions of (2) can be mapped into solutions of (1) by the (invertible) transformation

$$\psi_n(t) = \frac{1}{\sqrt{\mu}}\exp\left\{\frac{i\nu t}{\mu}\right\}u_n(t), \qquad \beta = \frac{\nu}{\mu}.$$

The aim of the present Letter is to study breather–breather collisions in the saturable DNLS equation (1) and to compare the results with those obtained in the continuous SNLS and the discrete cubic equation.

2. Numerical results

This model (1) has two conserved quantities: the Hamiltonian $H = \sum_n[\beta\log(1+|u_n|^2) + |u_{n-1} - u_n|^2]$ and the power (or norm) $P = \sum_n |u_n|^2$.

In order to reduce the dimension of the large parameter space to be considered, we have fixed β to $\beta = 2$. Higher values of β lead to solutions that only can be moved for a restricted set of power values [13]. Note that the localized stationary breather solution of [17] only exists for $\beta > 2$, and hence are not relevant to our discussions which focus on the $\beta = 2$ case. It would be interesting to extend the calculations in this Letter to other values of β to see if the presence of these stationary solutions affected the results given here.

A moving breather $v_n(t)$ is obtained by adding a thrust q to a stationary breather u_n, so that:

$$v_n(0) = u_n\exp(iqn). \qquad (3)$$

Notice that this procedure of obtaining moving breathers is similar to the marginal mode method introduced in [20,21] for Klein–Gordon lattices.

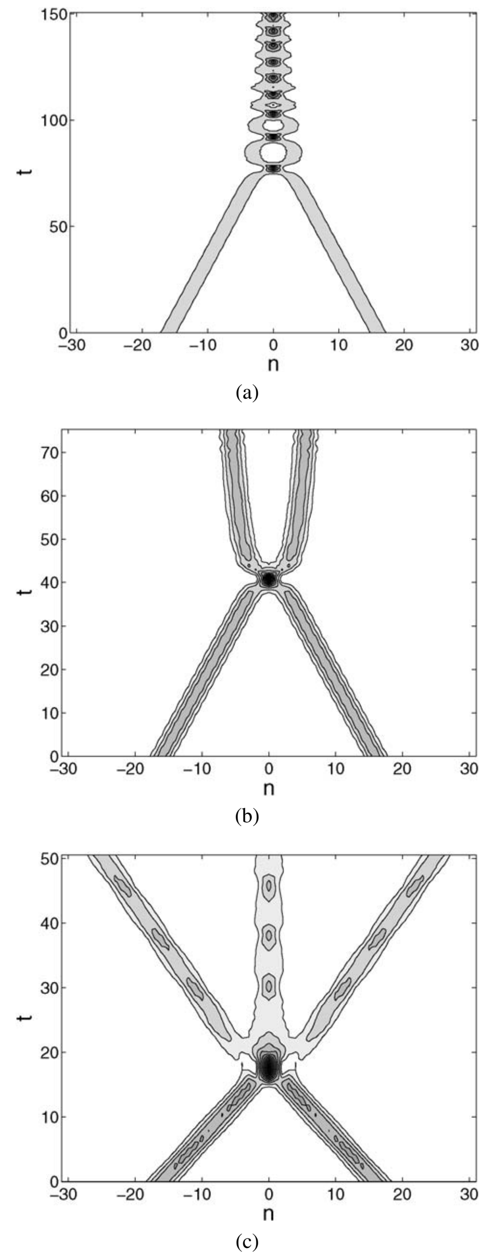

Fig. 1. Typical power density plots for (a) bound state formation ($P = 10$, $q = 0.1$), (b) reflection ($P = 10$, $q = 0.2$), and (c) breather creation ($P = 70$, $q = 0.5$). In all cases, OS collisions are considered, although these pictures do not vary considerably for IS collisions.

In the following, we consider the collision of two identical breathers moving in opposite directions with the same thrust q. Analogously to Ref. [10], we consider both inter-site (IS) and on-site (OS) collisions.

The collision scenario we observe for small P is quite simple: there exists a critical value q_c below which breathers form a bound state, and above which, breathers are reflected (see Fig. 1(a), (b) for examples, of these two cases). It can be observed that the bound state “oscillates” after the collision. The amplitude of these oscillations decreases when approaching to the critical point, whereas their “period” increases. (Note that the “reflection” case could equally be regarded as a transmission case as the two breathers are indistinguishable. In the case