Available online at www.sciencedirect.com
SCIENCE DIRECT®

Physics Letters A 357 (2006) 61–65

PHYSICS LETTERS A

www.elsevier.com/locate/pla

Electromagnetic light rays in local dielectrics

V.A. De Lorenci, R. Klippert *

Instituto de Ciências Exatas, Universidade Federal de Itajubá, Av. BPS 1303 Pinheirinho, 37500-903 Itajubá, MG, Brazil

Received 13 October 2005; received in revised form 25 March 2006; accepted 6 April 2006

Available online 18 April 2006

Communicated by P.R. Holland

Abstract

An approach to deal with the limit of geometrical optics of electromagnetic waves which propagate in moving nonlinear local dielectric media in the context of Maxwellian electrodynamics is here developed in order to apply to quite general material media. Fresnel equations for the light rays are generically found, and its solutions are intrinsically obtained. The multi-refringence problem is addressed, and no more than four monochromatic polarization modes are found to propagate there.

PACS: 42.15.-i; 42.25.Dd; 42.25.Lc

1. Introduction

Maxwell theory naturally yields electromagnetic wave propagation phenomena. This problem is fully solved in vacuum, since those equations become linear. Inside material media, however, polarization and magnetization fields may lead to nonlinear differential equations which are more difficult to deal with. Light propagation is also studied in the context of nonlinear Lagrangians for electrodynamics [1,2], the vacuum properties of which being similar to those of material media considered here. Some time ago, electrodynamics in material media has been considered as a possible scenario to investigate analogue models for gravitational phenomena (see Ref. [3] and references therein). This work intends to obtain and discuss the general Fresnel equations which completely describe the propagation of light rays in a nonhomogeneous nonisotropic nonlinear moving local dielectric medium.

In order to gain generality, equations are written in a covariant form which accounts for electrodynamics under the influence of the gravitational field $g_{\mu\nu}$ in an arbitrary causal orientable 4-dimensional Lorentzian manifold (i.e., a causal spacetime). We adopt the metric with signature $(+,-,-,-)$. Although this approach makes Maxwell equations rather involved, it is shown here that the propagation of the field discontinuities looks just the same as if working in the flat $g_{\mu\nu}=\eta_{\mu\nu}$ case (for which we have the Levi-Civita tensor also given in Cartesian coordinates with $\eta^{0123}=+1$). Partial derivatives with respect to the spacetime coordinates are denoted by a comma, and the corresponding covariant derivatives with respect to $g_{\mu\nu}$ are denoted by a semi-colon. Square brackets enclosing any number of tensorial indices mean a complete anti-symmetrization in these indices, as in $X^{[\alpha\beta]\gamma} \doteq X^{\alpha\beta\gamma} - X^{\beta\alpha\gamma}$. We also adopt[1] the standard decomposition for the covariant gradient of the observer's normalized velocity vector congruence V^μ as

$$V^\mu{}_{;\nu} = \frac{1}{3}\theta h^\mu_\nu + \sigma^\mu{}_\nu + \omega^\mu{}_\nu + a^\mu V_\nu, \tag{1}$$

with all a^μ, h^μ_ν, $\sigma_{\mu\nu}=\sigma_{\nu\mu}$ (traceless), and $\omega_{\mu\nu}=-\omega_{\mu\nu}$ being spacelike tensors. Physically, θ is the expansion factor, $\sigma^\mu{}_\nu$ is the shear tensor, $\omega^\mu{}_\nu$ is the vorticity tensor, and a^μ is the acceleration vector of the congruence V^μ, while the tensor $h^\mu_\nu \doteq \delta^\mu_\nu - V^\mu V_\nu$ projects onto the spatial sections of

* Corresponding author.
E-mail addresses: delorenci@unifei.edu.br (V.A. De Lorenci), klippert@unifei.edu.br (R. Klippert).

[1] The congruence V^μ was explicitly introduced in order to have a simpler description of the motion v^μ of the material media in Section 2: the medium may have a simple motion despite being described by possibly nonsimply moving observers. This feature is impossible to achieve if the relative velocity is the only one being considered.

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2006.04.010

this observer. We write the electromagnetic tensors [4] of field strength $F^{\mu\nu} = V^{[\mu}E^{\nu]} - \eta^{\mu\nu}{}_{\alpha\beta}V^{\alpha}B^{\beta}$ and field excitation $P^{\mu\nu} = V^{[\mu}D^{\nu]} - \eta^{\mu\nu}{}_{\alpha\beta}V^{\alpha}H^{\beta}$. We also make use of the electromagnetic current source vector J^{μ} expressed in geometrical nonrationalized Heavyside units such that the velocity of light in vacuum is $c = 1$. Only monochromatic waves are considered here, avoiding then subtleties concerning the meaning of their velocity.

Maxwell equations for the electromagnetic fields are set in Section 2 for a generically moving dielectric medium. Section 3 presents the propagation of light rays inside smooth media as wave discontinuities in the realm of this theory, generally yielding Fresnel equations. Section 4 deals with Fresnel equation in its full generality, while in Section 5 light propagation over discontinuous media is worked out. Section 6 summarizes our results.

2. Moving media

All the electromagnetic properties of many dielectric materials at rest are encoded in the phenomenological constitutive relations which give $P^{\mu\nu}$ as a function of $F^{\alpha\beta}$ in the $3+1$ form $D^{\alpha} = \varepsilon^{\alpha}{}_{\beta}E^{\beta}$ and $H^{\alpha} = \mu^{\alpha}{}_{\beta}B^{\beta}$, where $\mu^{\alpha}{}_{\beta}$ is the inverse of the permeability of the medium. (The whole argument which follows this point can easily be broadened to encompass the general local constitutive relations $D^{\alpha} = \varepsilon^{\alpha}{}_{\beta}E^{\beta} + \tilde{\varepsilon}^{\alpha}{}_{\beta}B^{\beta}$ and $H^{\alpha} = \mu^{\alpha}{}_{\beta}B^{\beta} + \tilde{\mu}^{\alpha}{}_{\beta}E^{\beta}$ with some modifications in Eqs. (2), (3), and no formal changes at all thereafter.) When electrodynamics is to deal with moving media with velocity v^{μ}, however, the constitutive equations above require some generalization. We adopt the local expressions [5,6]

$$D^{\alpha} - \eta^{\alpha\beta\mu\nu}V_{\beta}v_{\mu}H_{\nu} = \varepsilon^{\alpha}{}_{\beta}\big(E^{\beta} - \eta^{\beta\lambda\mu\nu}V_{\lambda}v_{\mu}B_{\nu}\big), \tag{2a}$$

$$H^{\alpha} + \eta^{\alpha\beta\mu\nu}V_{\beta}v_{\mu}D_{\nu} = \mu^{\alpha}{}_{\beta}\big(B^{\beta} + \eta^{\beta\lambda\mu\nu}V_{\lambda}v_{\mu}E_{\nu}\big), \tag{2b}$$

where the coefficients $\varepsilon^{\alpha}{}_{\beta}$ and $\mu^{\alpha}{}_{\beta}$ may be dependent of position as well as of the field strengths. One notes that the projection of the velocity v^{μ} of the medium onto the observer's spatial section $v^{\mu} \to h^{\mu}_{\nu}v^{\nu}$ clearly leave Eq. (2) unchanged; thus v^{μ} can be taken, and it will henceforth be considered, as this last spacelike vector, with $v^2 \doteq -h_{\mu\nu}v^{\mu}v^{\nu}$. Let us introduce the kinematic susceptibilities

$$\varepsilon_{(1)}{}^{\alpha}{}_{\beta} \doteq \frac{h^{\alpha}_{\gamma} + v^{\alpha}v_{\gamma}}{1-v^2}\big(\varepsilon^{\gamma}{}_{\beta} - h^{[\gamma}_{\beta}\,\mu_{\nu}{}^{\rho}v^{\nu]}v_{\rho}\big), \tag{3a}$$

$$\varepsilon_{(2)}{}^{\alpha}{}_{\beta} \doteq \frac{h^{\alpha}_{\gamma} + v^{\alpha}v_{\gamma}}{1-v^2}\big(\eta^{\gamma\lambda\mu\nu}\mu_{\nu\beta} + \eta_{\beta}{}^{\lambda\mu\nu}\varepsilon^{\gamma}{}_{\nu}\big)V_{\lambda}v_{\mu}, \tag{3b}$$

$$\mu_{(1)}{}^{\alpha}{}_{\beta} \doteq \frac{h^{\alpha}_{\gamma} + v^{\alpha}v_{\gamma}}{1-v^2}\big(\mu^{\gamma}{}_{\beta} - h^{[\gamma}_{\beta}\,\varepsilon_{\nu}{}^{\rho}v^{\nu]}v_{\rho}\big), \tag{3c}$$

$$\mu_{(2)}{}^{\alpha}{}_{\beta} \doteq \frac{h^{\alpha}_{\gamma} + v^{\alpha}v_{\gamma}}{v^2-1}\big(\eta^{\gamma\lambda\mu\nu}\varepsilon_{\nu\beta} + \eta_{\beta}{}^{\lambda\mu\nu}\mu^{\gamma}{}_{\nu}\big)V_{\lambda}v_{\mu}, \tag{3d}$$

which encompass both the electromagnetic behavior of the medium as well as its kinematics, as seen from the observer V^{μ}. All the four kinematic susceptibilities above may depend on all quantities E^{μ}, B^{μ}, v^{μ}, and x^{μ}. Simple algebra then yields the constitutive Eqs. (2) in the form

$$D^{\alpha} = \varepsilon_{(1)}{}^{\alpha}{}_{\beta}E^{\beta} + \varepsilon_{(2)}{}^{\alpha}{}_{\beta}B^{\beta}, \tag{4a}$$

$$H^{\alpha} = \mu_{(1)}{}^{\alpha}{}_{\beta}B^{\beta} + \mu_{(2)}{}^{\alpha}{}_{\beta}E^{\beta}. \tag{4b}$$

Eqs. (4) encode all the electromagnetically relevant properties of local dielectric media. Let us now define the following auxiliary 3-dimensional matrices:

$$d_{(1)}{}^{\alpha}{}_{\beta} \doteq \varepsilon_{(1)}{}^{\alpha}{}_{\beta} + \frac{\partial\varepsilon_{(1)}{}^{\alpha}{}_{\tau}}{\partial E^{\beta}}E^{\tau} + \frac{\partial\varepsilon_{(2)}{}^{\alpha}{}_{\tau}}{\partial E^{\beta}}B^{\tau}, \tag{5a}$$

$$d_{(2)}{}^{\alpha}{}_{\beta} \doteq \varepsilon_{(2)}{}^{\alpha}{}_{\beta} + \frac{\partial\varepsilon_{(1)}{}^{\alpha}{}_{\tau}}{\partial B^{\beta}}E^{\tau} + \frac{\partial\varepsilon_{(2)}{}^{\alpha}{}_{\tau}}{\partial B^{\beta}}B^{\tau}, \tag{5b}$$

$$h_{(1)}{}^{\alpha}{}_{\beta} \doteq \mu_{(1)}{}^{\alpha}{}_{\beta} + \frac{\partial\mu_{(1)}{}^{\alpha}{}_{\tau}}{\partial B^{\beta}}B^{\tau} + \frac{\partial\mu_{(2)}{}^{\alpha}{}_{\tau}}{\partial B^{\beta}}E^{\tau}, \tag{5c}$$

$$h_{(2)}{}^{\alpha}{}_{\beta} \doteq \mu_{(2)}{}^{\alpha}{}_{\beta} + \frac{\partial\mu_{(1)}{}^{\alpha}{}_{\tau}}{\partial E^{\beta}}B^{\tau} + \frac{\partial\mu_{(2)}{}^{\alpha}{}_{\tau}}{\partial E^{\beta}}E^{\tau}. \tag{5d}$$

Maxwell equations can be written either in compact form as $P^{\mu\nu}{}_{;\nu} = 4\pi J^{\mu}$ and $\eta^{\mu\nu\lambda\rho}F_{\mu\nu;\lambda} = 0$, or explicitly in terms of the $3+1$ electromagnetic field strengths E^{μ} and B^{μ} with the aid of Eqs. (5) as

$$B^{\alpha}{}_{;\alpha} + a_{\alpha}B^{\alpha} - \eta^{\alpha\beta\mu\nu}V_{\beta}\omega_{\mu\nu}E_{\alpha} = 0, \tag{6a}$$

$$\begin{aligned} &h^{\mu}_{\alpha}B^{\alpha}{}_{;\beta}V^{\beta} - \eta^{\mu\alpha\beta\lambda}V_{\alpha}E_{\beta;\lambda} \\ &\quad + \left(\frac{2\theta}{3}h^{\mu}_{\alpha} - \sigma^{\mu}{}_{\alpha} - \omega^{\mu}{}_{\alpha}\right)B^{\alpha} = 0, \end{aligned} \tag{6b}$$

$$\begin{aligned} &\big(\varepsilon_{(1)}{}^{\nu}{}_{\beta;\nu}E^{\beta} + \varepsilon_{(2)}{}^{\nu}{}_{\beta;\nu}B^{\beta}\big) \\ &\quad + \left(\frac{\partial\varepsilon_{(1)}{}^{\alpha}{}_{\beta}}{\partial v^{\mu}}E^{\beta} + \frac{\partial\varepsilon_{(2)}{}^{\alpha}{}_{\beta}}{\partial v^{\mu}}B^{\beta}\right)v^{\mu}{}_{;\alpha} \\ &\quad + d_{(1)}{}^{\nu}{}_{\beta}E^{\beta}{}_{;\nu} + d_{(2)}{}^{\nu}{}_{\beta}B^{\beta}{}_{;\nu} + \big(\varepsilon_{(1)}{}^{\alpha}{}_{\beta}E^{\beta} + \varepsilon_{(2)}{}^{\alpha}{}_{\beta}B^{\beta}\big)a_{\alpha} \\ &\quad + \eta_{\lambda}{}^{\nu\alpha\beta}V_{\nu}\omega_{\alpha\beta}\big(\mu_{(1)}{}^{\lambda}{}_{\beta}B^{\beta} + \mu_{(2)}{}^{\lambda}{}_{\beta}E^{\beta}\big) = 4\pi J^{\nu}V_{\nu}, \end{aligned} \tag{6c}$$

$$\begin{aligned} &h^{\mu}_{\alpha}\Bigg[\big(\varepsilon_{(1)}{}^{\alpha}{}_{\beta;\nu}E^{\beta} + \varepsilon_{(2)}{}^{\alpha}{}_{\beta;\nu}B^{\beta}\big) \\ &\quad + \left(\frac{\partial\varepsilon_{(1)}{}^{\alpha}{}_{\beta}}{\partial v^{\lambda}}E^{\beta} + \frac{\partial\varepsilon_{(2)}{}^{\alpha}{}_{\beta}}{\partial v^{\lambda}}B^{\beta}\right)v^{\lambda}{}_{;\nu} \\ &\quad + d_{(1)}{}^{\alpha}{}_{\beta}E^{\beta}{}_{;\nu} + d_{(2)}{}^{\alpha}{}_{\beta}B^{\beta}{}_{;\nu}\Bigg]V^{\nu} \\ &\quad + \eta^{\mu\nu\gamma}{}_{\alpha}V_{\gamma}\Bigg[\big(\mu_{(1)}{}^{\alpha}{}_{\beta;\nu}B^{\beta} + \mu_{(2)}{}^{\alpha}{}_{\beta;\nu}E^{\beta}\big) \\ &\quad + \left(\frac{\partial\mu_{(1)}{}^{\alpha}{}_{\beta}}{\partial v^{\lambda}}B^{\beta} + \frac{\partial\mu_{(2)}{}^{\alpha}{}_{\beta}}{\partial v^{\lambda}}E^{\beta}\right)v^{\lambda}{}_{;\nu} \\ &\quad + h_{(1)}{}^{\alpha}{}_{\beta}B^{\beta}{}_{;\nu} + h_{(2)}{}^{\alpha}{}_{\beta}E^{\beta}{}_{;\nu}\Bigg] \\ &\quad + \left(\frac{2\theta}{3}h^{\mu}_{\alpha} - \sigma^{\mu}{}_{\alpha} - \omega^{\mu}{}_{\alpha}\right)\big(\varepsilon_{(1)}{}^{\alpha}{}_{\beta}E^{\beta} + \varepsilon_{(2)}{}^{\alpha}{}_{\beta}B^{\beta}\big) \\ &\quad = -4\pi J^{\nu}h^{\mu}_{\nu}. \end{aligned} \tag{6d}$$

In Eqs. (6) it was already taken into account explicitly the eventual dependence there may be between the variables the kinematic susceptibilities depend upon. Thus, the covariant derivative $\varepsilon_{(1)}{}^{\alpha}{}_{\beta;\nu}$ is meant to be taken with constant E^{μ}, B^{μ}, and v^{μ}; and similarly for the others. Eqs. (6) are then expected to completely describe electromagnetic phenomena inside general