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A further improved extended Fan sub-equation method and its application to the (3 + 1)-dimensional Kadomstev–Petviashvili equation

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Abstract

A further improved extended Fan sub-equation method is proposed to seek more types of exact solutions of non-linear partial differential equations. Applying this method to the (3 + 1)-dimensional Kadomstev–Petviashvili equation, we obtain many new and more general non-travelling wave solutions including soliton-like solutions, triangular-like solutions, single and combined non-degenerate Jacobi elliptic wave function-like solutions, Weierstrass elliptic doubly-like periodic solutions.

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1. Introduction

In recent years, non-linear partial differential equations (NLPDEs) are widely used to describe many important phenomena and dynamic processes in physics, mechanics, chemistry, biology, etc. As mathematical models of the phenomena, the investigation of exact solutions of NLPDEs reveals to be very important for the understanding of these physical problems. Many mathematicians and physicists have well understood this importance when they decided to pay special attention to the development of sophisticated methods for constructing exact solutions of NLPDEs. A number of powerful methods have been presented, such as inverse scattering method [1], Hirota's bilinear method [2], Bäcklund transformation [3], Painlevé expansion [4], sine–cosine method [5], homotopy perturbation method [6], Adomian Padé approximation [7], homogenous balance method [8], variational method [9], tanh function method [10], Jacobi elliptic function expansion method [11], F-expansion method [12] and so on.

Recently, Fan [13] presented a new algebraic method to seek more solitary wave solutions of NLPDEs. This method was applied to many equations [14–16]. Later, Chen et al. [17], Jiao et al. [18] and Yomba [19] improved this method in different manners and obtained new formal solutions of some NLPDEs. Very recently, Yomba [20] proposed a modified extended Fan sub-equation method and obtained many general solutions of the (2 + 1)-dimensional Broer–Kaup–Kupershmidt equations.

In this Letter, we further improve the modified extended Fan sub-equation method [20] by making a more general transformation to seek more types of exact solutions of NLPDEs. In Section 2, we present a further improved extended Fan sub-equation method. In Section 3, we apply the new presented method to the (3 + 1)-dimensional Kadomstev–Petviashvili (KP) equation. In Section 4, some conclusions are given.

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2. Further improved extended Fan sub-equation method

For a given NLPDE with independent variables $x=(t,x_1,x_2,\dots,x_m)$ and dependent variable u:

$$F(u,u_t,u_{x_1},u_{x_2},\dots,u_{x_m},u_{x_1t},u_{x_2t},\dots,u_{x_mt},u_{tt},u_{x_1x_1},u_{x_2x_2},\dots,u_{x_mx_m},\dots)=0, \tag{1}$$

we seek its solutions in the new and more general form:

$$u=a_0+\sum_{i=1}^{n}\left\{a_i\phi^{-i}(\xi)+b_i\phi^{i}(\xi)+c_i\phi^{i-1}(\xi)\sqrt{\sum_{\rho=0}^{4}h_\rho\phi^\rho(\xi)}+d_i\phi^{-i}(\xi)\sqrt{\sum_{\rho=0}^{4}h_\rho\phi^\rho(\xi)}\right\}, \tag{2}$$

with $\phi(\xi)$ satisfying the following general elliptic equation:

$$\phi'^2=\left(\frac{d\phi}{d\xi}\right)^2=h_0+h_1\phi+h_2\phi^2+h_3\phi^3+h_4\phi^4, \tag{3}$$

where $a_0=a_0(x)$, $a_i=a_i(x)$, $b_i=b_i(x)$, $c_i=c_i(x)$, $d_i=d_i(x)$ $(i=1,2,\dots,n)$ and $\xi=\xi(x)$ are all functions to be determined later; h_0, h_1, h_2, h_3 and h_4 are constants. To determine u explicitly, we take the following four steps:

Step 1. Determine the integer n by balancing the highest order non-linear term(s) with the highest order partial derivatives of u in Eq. (1).

Step 2. Substitute (2) along with (3) into Eq. (1) and set all the coefficients of $\phi^j(\sqrt{\sum_{\rho=0}^{4}h_\rho\phi^\rho})^l$ $(l=0,1;\ j=0,\pm1,\pm2,\dots)$ to zero to derive a set of over-determined partial differential equations for a_0, a_i, b_i, c_i, d_i and ξ.

Step 3. Solve the over-determined partial differential equations with the aid of *Mathematica* and using Wu elimination method, we can obtain the explicit expressions for a_0, a_i, b_i, c_i, d_i and ξ.

Step 4. By using the results obtained in the above steps, we can derive a series of fundamental solutions of Eq. (1) depending on the different values of h_0, h_1, h_2, h_3 and h_4. By considering the different values of h_0, h_1, h_2, h_3 and h_4, Eq. (3) has many kinds of solutions which are listed in Ref. [20], where $\phi_{16}^{\mathrm{IV}}=\operatorname{sn}\xi\pm\mathrm{i}\operatorname{cs}\xi$ should be replaced by $\phi_{16}^{\mathrm{IV}}=\operatorname{sn}\xi\pm\mathrm{i}\operatorname{cn}\xi$.

3. Application to the (3 + 1)-dimensional KP equation

For the $(3+1)$-dimensional KP equation:

$$u_{xt}+6u_x^2+6uu_{xx}-u_{xxxx}-u_{yy}-u_{zz}=0. \tag{4}$$

Wang and Lou [21] and Senthilvelan [22] have studied its Painlevé property and special type of exact solutions. Hu [15] has obtained some travelling wave solutions by applying Fan algebraic method. Xie et al. [23] have obtained many non-travelling wave solutions by using an improved tanh function method. In this section, we apply our method to seek more types of non-travelling wave solutions.

According to Step 1 we can get $n=2$ for u, to search for explicit solutions of Eq. (4), we set $a_0=a_0(y,z,t)$, $a_1=a_1(y,z,t)$, $a_2=a_2(y,z,t)$, $b_1=b_1(y,z,t)$, $b_2=b_2(y,z,t)$, $c_1=c_1(y,z,t)$, $c_2=c_2(y,z,t)$, $d_1=d_1(y,z,t)$, $d_2=d_2(y,z,t)$, $\eta=\eta(y,z,t)$, $\xi=kx+\eta$, k is a non-zero constant, then we have

$$\begin{aligned}u={}&a_0+a_1\phi^{-1}(\xi)+a_2\phi^{-2}(\xi)+b_1\phi(\xi)+b_2\phi^2(\xi)+c_1\sqrt{\sum_{\rho=0}^{4}h_\rho\phi^\rho(\xi)}+c_2\phi(\xi)\sqrt{\sum_{\rho=0}^{4}h_\rho\phi^\rho(\xi)}\\&+d_1\phi^{-1}(\xi)\sqrt{\sum_{\rho=0}^{4}h_\rho\phi^\rho(\xi)}+d_2\phi^{-2}(\xi)\sqrt{\sum_{\rho=0}^{4}h_\rho\phi^\rho(\xi)}.\end{aligned} \tag{5}$$

With the aid of *Mathematica*, substituting (5) along with Eq. (3) into Eq. (4) yields a set of algebraic equations for $\phi^j(\sqrt{\sum_{\rho=0}^{4}h_\rho\phi^\rho})^l$ $(l=0,1;\ j=\pm1,\pm2,\dots)$. Setting each coefficient of $\phi^j(\sqrt{\sum_{\rho=0}^{4}h_\rho\phi^\rho})^l$ to zero, we can deduce a set of over-determined partial differential equations for a_0, a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2 and η. Solving these over-determined partial differential equations, we get six cases and only list two cases under the condition of $a_1\neq0$, $a_2\neq0$ and $d_2\neq0$ as following:

Case 1

$$a_0=\frac{k^4h_2\pm6k^4\sqrt{h_0h_4}+f_1^2(t)+f_2^2(t)-k(yf_1'(t)+zf_2'(t)+f_3'(t))}{6k^2}, \tag{6}$$

$$a_1=\frac{k^2h_1}{2},\qquad a_2=k^2h_0,\qquad b_1=\frac{k^2h_3}{2},\qquad b_2=k^2h_4,\qquad c_1=\pm k^2\sqrt{h_4}, \tag{7}$$