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Quantum Arnol'd diffusion in a rippled waveguide

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Abstract

We study the quantum Arnol'd diffusion for a particle moving in a quasi-1D waveguide bounded by a periodically rippled surface, in the presence of the time-periodic electric field. It was found that in a deep semiclassical region the diffusion-like motion occurs for a particle in the region corresponding to a stochastic layer surrounding the coupling resonance. The rate of the quantum diffusion turns out to be less than the corresponding classical one, thus indicating the influence of quantum coherent effects. Another result is that even in the case when such a diffusion is possible, it terminates in time due to the mechanism similar to that of the dynamical localization. The quantum Arnol'd diffusion represents a new type of quantum dynamics, and may be experimentally observed in measurements of a conductivity of low-dimensional mesoscopic structures.

As is well known, one of the mechanisms of the dynamical chaos in the Hamiltonian systems is due to the interaction between nonlinear resonances [1]. When the interaction is strong, this leads to the so-called global chaos which is characterized by a chaotic region spanned over the whole phase space of a system, although large isolated islands of stability may persist. For a weak interaction, the chaotic motion occurs only in the vicinity of separatrices of the resonances, in accordance with the Kolmogorov–Arnol'd–Moser (KAM) theory (see, for example, Ref. [2]). In the case of two degrees of freedom ($N = 2$), the passage of a trajectory from one stochastic region to another is blocked by KAM surfaces.

The situation changes drastically in many-dimensional ($N > 2$) systems for which the KAM surfaces no longer separate stochastic regions surrounding different resonances, and chaotic layers of the destroyed separatrices form a stochastic web that can cover the whole phase space. Thus, if the trajectory starts inside the stochastic web, it can diffuse throughout the phase space. This weak diffusion *along* stochastic webs was predicted by Arnol'd in 1964 [3], and since that time it is known as a very peculiar phenomenon, however, universal for many-dimensional nonlinear Hamiltonian systems (see, for example, review [4] and references therein).

Recently, much attention has been paid to the chaotic dynamics of a particle in a rippled channel (see, for example, Refs. [5–9]). The main interest was in the quantum-classical correspondence for the conditions of a strong chaos. Specifically, in Ref. [5] the transport properties of the channel in a ballistic regime were under study. Energy band structure, the structure of eigenfunctions and density of states have been calculated in [6,7]. The quantum states in the channel with rough boundaries, as well as the phenomena of quantum localization have been analyzed in [9]. The influence of an external magnetic field for narrow channels was investigated in [8]. These studies may have a direct relevance to the experiments with periodically modulated conducting channels. In this connection one can mention the investigation [10] of transport properties of a mesoscopic structure (sequence of quantum dots) with the periodic potential formed by metallic gates.

In contrast with the previous studies, below we address the regime of weak quantum chaos that occurs along the nonlinear resonances in the presence of an external time-periodic electric field. Our goal is to study the properties of the quantum Arnol'd diffusion which may be observed experimentally. The approach we use is based on the theory developed in Ref. [11] by making

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use of a simple model of two coupling nonlinear oscillators, one of which is driven by two-frequency external field.

We study the Arnol'd diffusion in a periodic quasi-one-dimensional waveguide with the upper profile given in dimensionless variables by the function $y = d + a\cos x$. Here x and y are the longitudinal and transverse coordinates, d is the average width, and a is the ripple amplitude. The low profile is assumed to be flat, $y = 0$. The nonlinear resonances arise due to the coupling between two degrees of freedom, with the resonance condition,

$$\eta = \frac{T_x}{T_y} = \frac{\omega_y}{\omega_x}. \tag{1}$$

Here T_y is the period of a transverse oscillation inside the channel, T_x is the time of flight of a particle over one period of the waveguide, ω_x and ω_y are the corresponding frequencies, and η is the rational number.

The mechanism of the classical Arnol'd diffusion in this system is illustrated in Fig. 1. Here some of the resonance lines for different values of η are shown on the ω_x-, ω_y-plane. Also, the curve of a constant kinetic energy $E = (p_x^2 + p_y^2)/2m$ is shown, determined by the equation

$$\omega_x^2 + \left(\frac{\omega_y d}{\pi}\right)^2 = \frac{2E}{m}. \tag{2}$$

Here E and m are the kinetic energy and particle mass, respectively, which we set to unity in what follows. The neighboring coupling resonances are isolated one from another by the KAM-surfaces, therefore, for a weak perturbation the transition between their stochastic layers is forbidden. Such a transition could occur in the case of the resonance overlap, i.e. in the case of the global chaos only. In the absence of an external field, the passage of a trajectory along any stochastic layer (this direction is shown at Fig. 1 by two arrows) is also impossible because of the energy conservation. However, the external time-periodic field removes the latter restriction, and a slow diffusion along stochastic layers becomes possible.

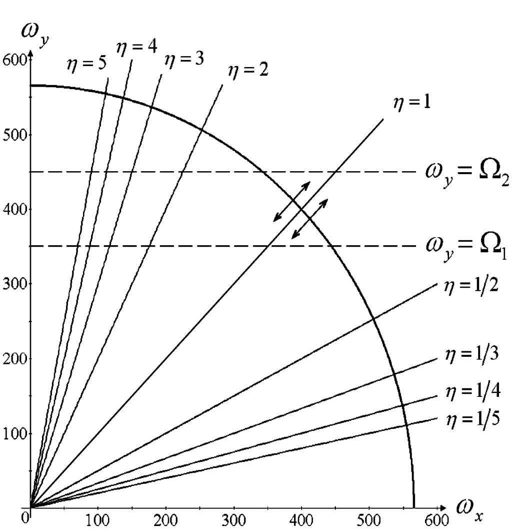

Fig. 1. Some of the coupling resonances for different values of η, with the isoenergetic curve $E = 1.6 \times 10^5$ in the frequency plane (for $d = \pi$ and $m = 1$). The locations of resonances are shown by dashed lines.

An external electric field with the corresponding potential $V(y,t) = -f_0 y(\cos \Omega_1 t + \cos \Omega_2 t)$ gives rise to two main resonances, $\omega_y = \Omega_1$ and $\omega_y = \Omega_2$. In order to calculate the diffusion rate, we consider a part of the Arnol'd stochastic web created by three resonances, namely, by the coupling resonance $\omega_x = \omega_y$ and two guiding resonances with frequencies Ω_1 and Ω_2. Therefore, we chose the initial conditions inside the stochastic layer of the coupling resonance. To avoid the overlapping of main resonances and provide a chaotic motion in the near-separatrix region, we assume the relation $a/f_0 = 10^{-3} \ll 1$ is fulfilled. In such a way, the regime of weak chaos is realized.

For the further analysis it is convenient to pass to the curvilinear coordinates x_i' in which both boundaries are flat [12]. The covariant coordinate representation of the Schrödinger equation has the following form,

$$-\frac{1}{2\sqrt{g}}\frac{\partial}{\partial x_i'}\sqrt{g}\,g_{ij}\frac{\partial \psi}{\partial x_j'} = E\psi \tag{3}$$

where g_{ik} is the metric tensor and $g \equiv \det(g_{ij})$. Here we use the units in which the Plank's constant and effective mass are equal to unity. As a result, the new coordinates are

$$x' = x, \qquad y' = \frac{y}{1 + \epsilon\cos x} \tag{4}$$

where $\epsilon = a/d$. In these coordinates the boundary conditions are $\psi(x',0) = \psi(x',d) = 0$ and the metric tensor is

$$g_{ij} = \begin{pmatrix} 1 & \frac{\epsilon x' \sin x'}{1+\epsilon\cos x'} \\ \frac{\epsilon x' \sin x'}{1+\epsilon\cos x'} & \frac{1+\epsilon^2 x'^2 \sin^2 x'}{(1+\epsilon\cos x')^2} \end{pmatrix}, \tag{5}$$

with the orthonormality condition,

$$\int \psi_i^\star \psi_j \sqrt{g}\, dS' = \delta_{ij}. \tag{6}$$

If the ripple amplitude a is small compared to the channel width d, one can safely keep only the first-order terms in ϵ in the Schrödinger equation (3). This strongly simplifies numerical simulations without the loss of generality. However, one should note that with an increase of roughness in the scattering profile $y(x)$ this approximation may be invalid, due to an influence of the so-called gradient scattering, see details in Ref. [9]. As a result, we obtain the following Hamiltonian [12],

$$\hat{H} = \hat{H}_0(x,y) + \hat{U}(x,y), \tag{7}$$

where

$$\hat{H}_0 = -\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \tag{8}$$

and

$$\hat{U} = \frac{\epsilon}{2}\left(2\cos x\frac{\partial^2}{\partial y^2} - 2y\sin x\frac{\partial^2}{\partial x \partial y} - y\cos x\frac{\partial}{\partial y} - \frac{1}{2}\cos x - \sin x\frac{\partial}{\partial x}\right). \tag{9}$$

Here and below we omitted primes in coordinates x' and y'.

Since the Hamiltonian is periodic in the longitudinal coordinate x, the eigenstates are Bloch states characterized by the