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A class of bound entangled states

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Abstract

We construct a set of PPT (positive partial transpose) states and show that these PPT states are not separable, thus present a class of bound entangled quantum states.

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Quantum entangled states are used as key resources in quantum information processing such as quantum teleportation, cryptography, dense coding, error correction and parallel computation [1,2]. To quantify the degree of entanglement a number of entanglement measures have been proposed for bipartite states. However most proposed measures of entanglement involve extremizations which are difficult to handle analytically. It turns out that to verify the separability of a general mixed states could be extremely difficult.

Among the quantum entangled states, there is a special kind of states that cannot be distilled. These states are called bound entangled states. Many powerful separability criteria could not detect the entanglement of these states, e.g. the bound entangled states given in [3–5]. A few new bound entangled states have been found recently by using the method of positive maps [6].

It has been shown [7] that any state which is entangled and satisfies positive partial transpose (PPT) condition [8] is not distillable. The existence of PPT entangled states was discussed in [9] and explicit examples were provided in [5], based on an elegant separability (range) criterion.

In [10] a special class of quantum states (d-computable states) were constructed. The entanglement of formation of these states can be analytically calculated and it turns out that all the states are entangled. In this Letter according to the construction of the d-computable states, we present first a class of PPT states, then by using the range criterion we prove that they are entangled.

Let $\mathcal{H}$ be an N-dimensional complex Hilbert space with orthonormal basis e_i, $i = 1, \ldots, N$. A general bipartite pure state on $\mathcal{H} \otimes \mathcal{H}$ is of the form,

$$|\psi\rangle = \sum_{i,j=1}^{N} a_{ij} e_i \otimes e_j, \quad a_{ij} \in \mathcal{C} \tag{1}$$

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with normalization $\sum_{i,j=1}^{N} a_{ij}a_{ij}^* = 1$. Let A denote the matrix with entries given by a_{ij} in (1). Set

$$A = \begin{bmatrix} 0 & b_1 & a & -c \\ -b_1 & 0 & c & d \\ -a & -c & 0 & -c_1 \\ c & -d & c_1 & 0 \end{bmatrix}, \tag{2}$$

where $a, c, d, b_1, c_1 \in \mathcal{C}$.

It was shown that all pure states $|\psi\rangle$ with A given by (2) have a simple formula of the generalized concurrence C, such that the entanglement of formation is a monotonically increasing function of C [10]. Moreover the entanglement of formation of all mixed states ρ with decompositions on pure states with A given by (2) can be analytically calculated. As all the states with A of (2) are entangled, these mixed states ρ are also entangled.

In fact A is an antisymmetric matrix. Any antisymmetric matrices are equivalent to the following standard form under similarity transformations:

$$A_1 = \begin{bmatrix} 0 & \lambda_1 & 0 & 0 \\ -\lambda_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 \\ 0 & 0 & -\lambda_2 & 0 \end{bmatrix} \quad \text{or} \quad A_2 = \begin{bmatrix} 0 & 0 & \lambda_1 & 0 \\ 0 & 0 & 0 & \lambda_2 \\ -\lambda_1 & 0 & 0 & 0 \\ 0 & -\lambda_2 & 0 & 0 \end{bmatrix}.$$

If we set $\lambda_1 = \pm b$, $\lambda_2 = -c$ in A_1, the matrix A_1 gives rise to two pure states

$$|\psi_{\pm b}\rangle = [0 \quad \pm b \quad 0 \quad 0 \quad \mp b \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -c \quad 0 \quad 0 \quad c \quad 0]^t,$$

and hence two projectors $\rho_{\pm b} = |\psi_{\pm b}\rangle\langle\psi_{\pm b}|$, where t denotes the transposition. We define

$$\rho_b = \frac{1}{2}\rho_{+b} + \frac{1}{2}\rho_{-b}.$$

And if we set $\lambda_1 = \pm a$, $\lambda_2 = d$ in A_2 we have

$$\rho_a = \frac{1}{2}\rho_{+a} + \frac{1}{2}\rho_{-a},$$

where $\rho_{\pm a} = |\psi_{\pm a}\rangle\langle\psi_{\pm a}|$,

$$|\psi_{\pm a}\rangle = [0 \quad 0 \quad \pm a \quad 0 \quad 0 \quad 0 \quad 0 \quad d \quad \mp a \quad 0 \quad 0 \quad 0 \quad 0 \quad -d \quad 0 \quad 0]^t.$$

We define

$$\rho_0 = \frac{1}{2}\rho_a + \frac{1}{2}\rho_b. \tag{3}$$

The state ρ_0 is not separable as its partial transposed matrix has a negative eigenvalue. Below we mix ρ_0 with some separable states in such a way that the resulting state will be both partial transposition positive and entangled. Let I_4 be a 16×16 matrix with only non-zero entries $(I_4)_{1,1} = (I_4)_{6,6} = (I_4)_{11,11} = (I_4)_{16,16} = 1$. We consider $\rho = (1-\varepsilon)I_4 + \varepsilon\rho_0$, which is of the form:

$$\rho = \begin{bmatrix}
x_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & x_3 & 0 & 0 & -x_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & x_2 & 0 & 0 & 0 & 0 & 0 & -x_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -x_3 & 0 & 0 & x_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & x_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & x_5 & 0 & 0 & 0 & 0 & 0 & -x_5 & 0 & 0 \\
0 & 0 & -x_2 & 0 & 0 & 0 & 0 & 0 & x_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_4 & 0 & 0 & -x_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -x_5 & 0 & 0 & 0 & 0 & 0 & x_5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x_4 & 0 & 0 & x_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_1
\end{bmatrix}, \tag{4}$$

where $x_1 = \frac{1-\varepsilon}{4}$, $x_2 = \frac{\varepsilon}{2}|a|^2$, $x_3 = \frac{\varepsilon}{2}|b|^2$, $x_4 = \frac{\varepsilon}{2}|c|^2$, $x_5 = \frac{\varepsilon}{2}|d|^2$.