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New results on stability analysis of neural networks with time-varying delays

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Abstract

In this Letter the time delay dependent stability problem is investigated for a class of time delay neural networks. By constructing novel Lyapunov Krasovskii functional, we propose the new stability results for time delay neural network. The sufficient conditions obtained in this Letter are looser than those in the former literature. Specially, our results include the time delay independent results obtained in some existing literature. The stability conditions are all in the form of linear matrix inequalities (LMIs), which can be computed and optimized easily. Finally, numerical examples are given to show the superiority of the main results.

Keywords: Time delay neural network; Stability analysis; Lyapunov Krasovskii functional

1. Introduction

In recent years, neural networks are widely studied, because of their immense potentials of application prospective. When designing a neural network to solve a problem such as linear program or pattern recognition, we need foremost to guarantee that the neural network model is globally asymptotically stable.

Global stability problem of various neural networks has been investigated extensively. Global asymptotic stability for time delay free neural network is studied in [1–4]. In practice, time delays are often encountered in various engineering, biological, and economical systems [5]. Due to the finite speed of information processing, the existence of time delays frequently causes oscillation, divergence, or instability in neural networks. In recent years, the stability problem of delayed neural networks has become a topic of great theoretic and practical importance. This issue has gained increasing interest in applications to signal and image processing, artificial intelligence, and so on. The stability results obtained on time delay neural networks in the existing literature are mainly time delay independent [6–14]. As we know time delay dependent results are looser than the time delay independent ones when the delays are small. So it is necessary to propose the time delay dependent stability results. In [15], the authors first propose the time delay dependent stability conditions for the time delay neural network. But the results obtained are difficult to compute the maximize delay constants.

In this Letter we consider the time delay dependent stability problem for a class of time delay neural networks. By employing new Lyapunov Krasovskii functional, we propose the novel sufficient conditions for the time delay neural network. The sufficient conditions obtained in this Letter are looser than those in the former literature. Specially, our stability results include the time delay independent ones in the former literature. The stability conditions obtained in this Letter are all in the form of LMIs. Finally, numerical examples will be given to show the effectiveness of the main results.

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2. System formulation

The dynamic behavior of a continuos time delay neural network can be described as follows

$$\dot{x}(t) = -Cx(t) + Af\big(x(t)\big) + Bf\big(x\big(t-d(t)\big)\big) + I, \tag{1}$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^T \in R^n$, $f(x) = [f_1(x_1(t)), \ldots, f_n(x_n(t))]^T$, $f(x(t-d(t))) = [f_1(x_1(t-d(t))), \ldots, f_n(x_n(t-d(t)))]^T$. $C = \mathrm{diag}(c_i)$ with $c_i > 0$, $A = \{a_{ij}\}$ is referred to as the feedback matrix, $B = \{b_{ij}\}$ represents the delayed feedback matrix, while $I = [I_1, I_2, \ldots, I_n]$ are the constant inputs. The delay considered is time-varying, which satisfies $0 \leqslant d(t) \leqslant d$ and $\dot{d}(t) \leqslant \mu$. In this Letter, we assume the activation functions $f_i(\cdot)$ $(i \in [1, n])$ are bounded and satisfy the following inequalities with $f_i(0) = 0$

$$0 \leqslant \frac{f_i(y_1) - f_i(y_2)}{y_1 - y_2} \leqslant h_i, \tag{2}$$

where h_i are positive scalars. This type of activation functions is clearly more general than both the usual sigmoid activation functions and the piecewise linear function: $f_i(x_i) = \frac{1}{2}(|x_i + 1| - |x_i - 1|)$.

The initial conditions associated with system (1) are as follows

$$x = \phi(s), \quad s \in [-d, 0],$$

where $\phi(s)$ is a continuous function vector.

Assume x^* is an equilibrium point of the Eq. (1), then we choose the coordinate transformation $z = x - x^*$. System (1) is changed into the following error system

$$\dot{z} = -Cz(t) + Ag\big(z(t)\big) + Bg\big(z\big(t-d(t)\big)\big), \tag{3}$$

where $g(z) = [g_1(z_1(t)), g_2(z_2(t)), \ldots, g_n(z_n(t))]$ and $g_i(z_i(t)) = f_i(z_i(t) + x_i^*) - f_i(x_i^*)$. According to inequality (2), one can obtain that

$$g_i^2\big(z_i(t)\big) \leqslant h_i z_i g_i\big(z_i(t)\big) \leqslant h_i^2 z_i^2. \tag{4}$$

From above analysis, we can see that the stability problem of system (1) on equilibrium x^* is changed into the zero stability problem of system (3). Therefore, in the following part we will investigate the stability analysis problem for system (3). Based on Lyapunov Krasovskii functional method, new stability conditions will be proposed.

3. Main results

First we consider the case that the bound of time derivative of the delay satisfies $\mu < 1$. We have the following stability theorem.

Theorem. *The system (3) is asymptotically stable if there exist positive matrices P, Q_1, Q_2, and positive diagonal matrices D_1 and D_2 such that the following LMIs hold*

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} \\ * & X_{22} & X_{23} & X_{24} & X_{25} \\ * & * & X_{33} & X_{34} & X_{35} \\ * & * & * & X_{44} & X_{45} \\ * & * & * & * & X_{55} \end{bmatrix} \geqslant 0, \tag{5}$$

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * & \Xi_{44} \end{bmatrix} < 0, \tag{6}$$

where $*$ represents the transpose of the corresponding matrix and

$$\begin{aligned}
\Xi_{11} &= -PC - C^T P + X_{15}^T + X_{15} + Q_1 + dC^T X_{55} C + dX_{11}, \\
\Xi_{12} &= X_{25}^T - X_{15} + dX_{12}, \\
\Xi_{13} &= PA + X_{35}^T + HD_1 - C^T L^T - dC^T X_{55} A + dX_{13}, \\
\Xi_{14} &= PB + X_{45}^T - dC^T X_{55} B + dX_{14}, \\
\Xi_{22} &= -X_{25} - X_{25}^T - (1-\mu) Q_1 + dX_{22}, \\
\Xi_{23} &= -X_{35}^T + dX_{23},
\end{aligned}$$