Available online at www.sciencedirect.com

SCIENCE DIRECT®

Physics Letters A 352 (2006) 386–397

PHYSICS LETTERS A

www.elsevier.com/locate/pla

Analysis and circuit implementation of a new 4D chaotic system ☆

Guoyuan Qi [a,*], Guanrong Chen [b]

[a] Department of Automation, Tianjin University of Science & Technology, Tianjin 300222, PR China

[b] Department of Electronic Engineering, City University of Hong Kong, Hong Kong SAR, PR China

Received 9 August 2005; received in revised form 13 December 2005; accepted 14 December 2005

Available online 27 December 2005

Communicated by A.P. Fordy

Abstract

This Letter contains three parts. First, it analyzes some basic properties of a new complex four-dimensional (4D) continuous autonomous chaotic system, in which each equation contains a cubic cross-product term. The new system has 9 equilibria, which display graceful symmetry with respect to the origin and the coordinate planes, and they have similarity associated with their linearized characteristics and along with invariant manifolds. Second, under constant control, the system displays (i) two coexisting symmetric double-wing chaotic attractors simultaneously, and (ii) two coexisting asymmetric double-wing and two coexisting single-wing attractors including chaotic, period-doubling, and periodic orbits. The evolution process of an attractor from double-wing to single-wing is investigated via a distribution diagram of equilibria and bifurcation diagrams of the system states. Finally, several circuits are built for different configurations of the new system, which show a good agreement between computer simulations and experimental results, revealing some important distinctions in applications arising from different frequencies used.

Keywords: Four-dimensional chaotic system; Lyapunov exponent; Double-wing chaotic attractor; Single-wing chaotic attractor; Pitchfork bifurcation; Hopf bifurcation; Circuit implementation

1. Introduction

Chaos has been intensively studied in the last four decades within the science, mathematics and engineering communications [1,2]. Generating or enhancing chaos is important in studying chaotic dynamics and their applications to encryption and communications, which are generally developed along two directions: one approach is generating multi-scroll chaotic attractors based on some existing chaotic systems such as generalized Chua's circuits by using some nonsmooth nonlinear functions such as piecewise-linear (PWL) functions [3,4], stair functions [5], hysteresis functions [6], and saturated functions [7], etc. Those nonlinearities do not contain quadratic terms, and the produced scrolls of chaotic attractors have cyclic shapes. It has been noticed that in the family of generalized Chua's circuits, periodic orbits rarely exist. Another approach is searching for some new three-dimensional (3D) chaotic attractors generated by systems like Chua's circuit with cubic nonlinearity [8] and generalized Lorenz systems with only quadratic nonlinearity. For example, Vaněček and Čelikovský [9] introduced the so-called generalized Lorenz system. More recently, Chen, Lü et al. [10–12,21] found some similar but nonequivalent chaotic systems. Furthermore, Čelikovský and Chen [13] presented a generalized Lorenz canonical form, including the hyperbolic-type, which covers a very large class of 3D quadratic autonomous chaotic systems. Generally, these generalized Lorenz systems display one single chaotic attractor with two butterfly wings, which is different from the attractors of generalized Chua's circuits at least in shape. Recently, Qi et al. [20] constructed a new 4D autonomous chaotic system, which has cubic cross-product nonlinearity in each equation. This system can generate complex dynamics within wide parameters ranges, including chaos, Hopf bifurcation, period-doubling bifurcation, periodic orbit, sink and source, and so on.

☆ This work was supported by a grant from the Research Grants Council of the Hong Kong SAR, China (CityU 1114/05E) and by a Grant from the Science and Technology Development Foundation of Tianjin Colleges (20051528).

* Corresponding author.

E-mail addresses: qi_gy580@sohu.com (G. Qi), eegchen@cityu.edu.hk (G. Chen).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2005.12.030

The main objective of this Letter is to carry out some basic analysis on the new 4D chaotic system, showing that it has 9 symmetric real equilibria with respect to the origin and the coordinate planes, respectively, which are divided into three types according to their similarity with respect to some linearization characteristics. The Letter also carries out some simulations, showing that the system can display: (i) two coexisting symmetric double-wing chaotic attractors, (ii) two coexisting asymmetric double-wing chaotic attractors and two single-wing chaotic attractors, respectively, under constant control. It is furthermore to investigate the evolution process from double-wing to single-wing via a distribution diagram of equilibria, and to analyze bifurcation diagrams of the system states. Finally, the Letter reports several circuits design, showing a good agreement between computer simulations and experimental results.

2. Some basic properties of the four-dimensional system

The 4D autonomous system is described by [20]

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_2x_3x_4,\\
\dot{x}_2 &= b(x_1 + x_2) - x_1x_3x_4,\\
\dot{x}_3 &= -cx_3 + x_1x_2x_4,\\
\dot{x}_4 &= -dx_4 + x_1x_2x_3,
\end{aligned} \tag{1}$$

where x_i $(i = 1, 2, 3, 4)$ are the state variables, and a, b, c, d are positive constant parameters.

2.1. Equilibria

To further analyze the system, a good place to start to find its equilibria, thereby characterizing the behaviors of solutions near these points based on the linearized system. The distribution and linearized characteristics of the equilibria influence the dynamics of the system. The equilibria of system (1) can be found by solving the following algebraic equations simultaneously:

$$\begin{aligned}
&a(x_2 - x_1) + x_2x_3x_4 = 0, \qquad b(x_1 + x_2) - x_1x_3x_4 = 0,\\
&-cx_3 + x_1x_2x_4 = 0, \qquad -dx_4 + x_1x_2x_3 = 0.
\end{aligned} \tag{2}$$

By calculation, one can find 9 real equilibria including zero, which can be classified into three kinds, and each kind has the same eigenvalues (see Remark 4). Let

$$\begin{aligned}
&q = \sqrt{cd}, \qquad p = \sqrt{a^2 + 6ab + b^2},\\
&g = p + a + b, \qquad h = p - a + b,\\
&m = p - a - b, \qquad n = p + a - b,\\
&x_1^1 = \sqrt{gq/(2a)}, \qquad x_2^1 = \sqrt{2aq/g},\\
&x_3^1 = \sqrt{hd/(2q)}, \qquad x_4^1 = \sqrt{hq/2d},
\end{aligned} \tag{3}$$

$$\begin{aligned}
&x_1^2 = \sqrt{mq/(2a)}, \qquad x_2^2 = \sqrt{2aq/m},\\
&x_3^2 = \sqrt{nd/(2q)}, \qquad x_4^2 = \sqrt{nq/(2d)}.
\end{aligned} \tag{4}$$

The first kind of nonzero equilibria includes

$$\begin{aligned}
&S_1 = [x_1^1, x_2^1, x_3^1, x_4^1], \qquad S_2 = [-x_1^1, -x_2^1, x_3^1, x_4^1],\\
&S_3 = [x_1^1, x_2^1, -x_3^1, -x_4^1], \qquad S_4 = [-x_1^1, -x_2^1, -x_3^1, -x_4^1].
\end{aligned} \tag{5}$$

The second kind of nonzero equilibria includes

$$\begin{aligned}
&S_5 = [x_1^2, x_2^2, x_3^2, x_4^2], \qquad S_6 = [-x_1^2, -x_2^2, x_3^2, x_4^2],\\
&S_7 = [x_1^2, x_2^2, -x_3^2, -x_4^2], \qquad S_8 = [-x_1^2, -x_2^2, -x_3^2, -x_4^2].
\end{aligned} \tag{6}$$

The third kind of equilibria is the zero equilibrium $S_0 = [0, 0, 0, 0]$.

Note that these equilibria have some symmetry with respect to origin or the coordinate planes, attributing to the symmetry of system (1).

2.2. Symmetry and invariance

Remark 1. System (1) is symmetric with respect to the coordinate plane x_3–x_4, which is easily proved via the following transformation:

$$(x_1, x_2, x_3, x_4) \to (-x_1, -x_2, x_3, x_4). \tag{7}$$

For convenience, use $S_{i,j}$ to denote the pair of S_i and S_j, $i, j = 1, \dots, 9$. From (5) and (6), equilibria $S_{1,2}$ and $S_{5,6}$ are symmetric pairs with respect to plane x_3–x_4, which correspond to the property stated in Remark 1.

Remark 2. System (1) is symmetric with respect to the coordinate plane x_1–x_2, which is proved via the following transformation:

$$(x_1, x_2, x_3, x_4) \to (x_1, x_2, -x_3, -x_4). \tag{8}$$

Similarly, there are two pairs of symmetric equilibria, $S_{3,4}$ and $S_{7,8}$ with respect to the x_1–x_2 plane.

Remark 3. System (1) is symmetric with respect to the origin, which is proved via the following transformation:

$$(x_1, x_2, x_3, x_4) \to (-x_1, -x_2, -x_3, -x_4). \tag{9}$$

It can be verified that S_i, $i = 1, \dots, 4$, are symmetric equilibria with respect to the origin, and do S_i, $i = 5, \dots, 8$.

2.3. Similarity

By linearizing system (1) at S_0, one obtains the Jacobian as follows:

$$A_0 = \begin{bmatrix} -a & a & 0 & 0 \\ b & b & 0 & 0 \\ 0 & 0 & -c & 0 \\ 0 & 0 & 0 & -d \end{bmatrix}. \tag{10}$$

The eigenvalues of matrix A_0 are

$$\begin{aligned}
&\lambda_{01} = \frac{1}{2}(b - a + p), \qquad \lambda_{02} = \frac{1}{2}(b - a - p),\\
&\lambda_{03} = -c, \qquad \lambda_{04} = -d,
\end{aligned} \tag{11}$$