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Diffraction efficiency of non-uniform gratings in a $Bi_{12}SiO_{20}$ crystal for a non-linear regime under an external d.c. field

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Abstract

We assess the influence of the non-uniformity of a recorded grating on the prediction of the diffraction efficiency taking into account optical activity, absorption of light, birefringence, polarization angle of the incident beam and an external d.c. field. The influence is noteworthy for samples larger than 3 mm.

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Due to their high photo refractive sensitivity for volume holographic grating formation, high response rate, long holographic storage times and unlimited recyclability the photo refractive crystals of the sillenite family $Bi_{12}MO_{20}$ (where M = Si, Ge or Ti) have been extensively studied for their potential use in device applications [1–3].

In particular the diffraction properties of $Bi_{12}MO_{20}$ (BSO) have been calculated by many authors. Several of them have proposed analytical or numerical solutions using a great variety of approaches considering optical activity, in thin crystals, assuming vector wave coupling, and using a spatially uniform grating. Other effects such as birefringence with d.c. and a.c. applied fields, diffusion dominated transport, and linear and circular polarized illuminations have also been considered [4–8]. Increasing the generality of the treatment, the problem has also been solved considering an arbitrary angle between the grating vector and the [001] axis, including the piezoelectric and elastooptic effects [9].

Nevertheless to our knowledge all the treatments have considered in one way or the other, a spatially uniform grating along the sample thickness. This is a reasonable assumption for weak coupling or thin crystals (< 3 mm), but not for thick samples or strong coupling which is needed to enhance the photo refractive response of BSO. Therefore it is important to know the behavior of crystal samples under these conditions which are important for practical applications. When the coupling between the two waves is considerable there is a spatial redistribution of the light intensity pattern. This redistribution results in changes of the light modulation across the crystal and therefore the grating is non-uniform and its amplitude and phase become a function of z.

In this work we consider steady transmission gratings and we asses the influence of the non-uniformity of a recorded grating on the prediction of the diffraction efficiency. We do this by comparing the results of the calculation of the diffraction efficiency of a uniform grating with the results obtained for a non-uniform grating along the sample thickness, in a BSO crystal (2.5 cm thick). We took into account vector wave coupling, optical activity, birefringence, absorption of light, large values of light modulation, different values for the applied electric field

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(0, 2.5, 5 and 10 kV/cm) under Bragg read out conditions and several polarization angles of the incident beams. We considered the two common optical configurations K_G // [001] and $K_G \perp$ [001]. Because of the symmetry in the crystal and the geometry we are considering here we can neglect elastooptical, piezoelectric, photo voltaic and photo galvanic effects [10–12].

First, we numerically solved the material rate equations [13–15], following a method described elsewhere [16]. This solution gives the full space charge field, E_{esc} and from this we obtained its fundamental Fourier component, E_1, and its phase, ϕ. This was done for all mentioned values of the applied field and several values of m_0 between 0 and 1 to obtain $E_1(m)$. Then, we considered the interaction of two monochromatic linearly polarized plane waves,

$$\vec{A}_1(z) = A_{1\xi}(z)\hat{u}_\xi + A_{1\varsigma}(z)\hat{u}_\varsigma, \tag{1}$$

$$\vec{A}_2(z) = A_{2\xi}(z)\hat{u}_\xi + A_{2\varsigma}(z)\hat{u}_\varsigma \tag{2}$$

that propagate inside the sample with two components: one perpendicular ($\hat{u}_\xi$) and the other parallel ($\hat{u}_\zeta$) to the plane of incidence (x–z). The light modulation, $m(z)$, in the sample is complex:

$$m(z) = \left[A_{1\xi}(z)A_{2\xi}(z)^* + A_{1\varsigma}(z)^*A_{2\varsigma}(z)\right]/I_0. \tag{3}$$

The polarization angle of an incident beam, Φ_p, is the angle between the polarization vector and the x axis.

The coupled beam equations for the paraxial and slow variation approximations for K_G // [001], taking into account birefringence, optical activity and absorption are [5]

$$\frac{dA_{1\varsigma}(z)}{dz} = -\rho A_{1\xi}(z) - \frac{\alpha}{2}A_{1\varsigma}(z), \tag{4a}$$

$$\frac{dA_{1\xi}(z)}{dz} = \rho A_{1\varsigma}(z) + i\kappa_0 A_{1\xi}(z) + i\kappa_1^*(z)A_{2\xi}(z) - \frac{\alpha}{2}A_{1\xi}(z), \tag{4b}$$

$$\frac{dA_{2\varsigma}(z)}{dz} = -\rho A_{2\xi}(z) - \frac{\alpha}{2}A_{2\varsigma}(z), \tag{4c}$$

$$\frac{dA_{2\xi}(z)}{dz} = \rho A_{2\varsigma}(z) + i\kappa_0 A_{2\xi}(z) + i\kappa_1(z)A_{1\xi}(z) - \frac{\alpha}{2}A_{2\xi}(z). \tag{4d}$$

Where ρ is the optical activity, α is the absorption constant and

$$\kappa_0 = \pi n_0^3 r E_0/(\lambda\cos\theta). \tag{5}$$

λ is the light wave length, θ is the incidence Bragg's angle; n_0 is the average refraction index; r is the electro-optic coefficient; E_0 is the d.c. applied external field and

$$\kappa_1(z) = \pi\Delta n_1(z)/(\lambda\cos\theta). \tag{6}$$

Notice that $\kappa_1(z)$ and the variation of the recorded refraction index, $\Delta n_1(z)$ are complex variables and

$$\Delta n_1(z) = n_0^3 r\left|E_1(z)\right|m(z)\exp\bigl(i\phi(z)\bigr)/\bigl(2\left|m(z)\right|\bigr). \tag{7}$$

The corresponding set of coupled wave equations for the $K_G \perp$ [001] orientation is [5]

$$\frac{dA_{1\xi}(z)}{dz} = (\rho - i\kappa_0)A_{1\varsigma}(z) - i\kappa_1^*(z)A_{2\varsigma}(z) - \frac{\alpha}{2}A_{1\xi}(z), \tag{8a}$$

$$\frac{dA_{1\varsigma}(z)}{dz} = -(\rho + i\kappa_0)A_{1\xi}(z) - i\kappa_1^*(z)A_{2\xi}(z) - \frac{\alpha}{2}A_{1\varsigma}(z), \tag{8b}$$

$$\frac{dA_{2\varsigma}(z)}{dz} = -(\rho + i\kappa_0)A_{2\xi}(z) - i\kappa_1(z)A_{1\xi}(z) - \frac{\alpha}{2}A_{2\varsigma}(z), \tag{8c}$$

$$\frac{dA_{2\xi}(z)}{dz} = (\rho - i\kappa_0)A_{2\varsigma}(z) - i\kappa_1(z)A_{1\varsigma}(z) - \frac{\alpha}{2}A_{2\xi}(z). \tag{8d}$$

We solved these sets of equations ((4) and (8)) with no restrictions on the parameters (ρ, k_0, k_1) and in a self-consisting way to take into account the variation with depth of the refraction index given by $\Delta n_1(z)$. We did this in the following manner. We divided the sample in thin layers of thickness Δz [17] in such a way that within each layer $k_1(z)$ is practically constant. In this way within each layer analytical solutions of the coupled equations were valid. When a small change (not larger than 0.1%) in this variable occurred, we calculated the new corresponding set of values of constants for the corresponding interval Δz. We started by using $\kappa_1(z=0)$ and evaluating the initial set of constants for the first layer at the surface of the sample. For the following layers, the values of the complex amplitudes of the beams at the end of each interval were used to evaluate the $m(z)$ and from this, $E_1(m(z))$ to obtain the new value of κ_1 at z_0, and the set of new constants for the next layer. We performed this calculation for the grating recording assuming that the two beams enter the crystal with identical polarization. Then we considered the reading stage, where the incident beam $\vec{A}_1 = \vec{A}_i$ couples itself with the diffracted beam $\vec{A}_2 = \vec{A}_d$ via the already written, and now fixed, refractive index with the boundary condition $\vec{A}_d(z=0) = 0$. The diffraction efficiency is

$$\eta(z) = I_i(z)/I_d(z=0).$$

In our calculations we considered a fringe spacing of 10 μm, $I_0 = 5$ mW/cm^2 (average light intensity); $m(z=0) = m_0 = 0.9$ and the usual physical parameters for BSO: $\varepsilon = 56$ (dielectric constant); $n_0 = 2.53$ (average refractive index); $r = 5.0 \times 10^{-12}$ m/V (electrooptic coefficient); $N_D = 10^{25}$ m^{-3} (donor density); $N_A = 10^{22}$ m^{-3} (acceptor density); $\mu = 3 \times 10^{-6}$ m^2/V s (mobility); $\gamma = 1.6 \times 10^{-17}$ m^3 s^{-1} (carrier recombination constant), and $\sigma = 1 \times 10^{-5}$ m^2/J (Photo ionization cross section). The writing process was done considering green light ($\lambda = 532$ nm, $\rho = 386$ degrees/cm and $\alpha = 0.65$ cm^{-1}). Reading was performed considering red light ($\lambda = 632$ nm, $\rho = 214$ degrees/cm and $\alpha = 0.3$ cm^{-1}) [5,16,18].

Our results are given as follows. In Fig. 1 we compare the light modulation assuming a uniform grating, curve (a), with the results obtained from the self consistent calculation (curves (b) and (c)), assuming a non-uniform grating. Here the grating was recorded for K_G // [001] and an applied field of 10 kV/cm. Curve (b) is with no absorption and curve (c) is with absorption. Notice that only for thin samples ($z < 1.0$ mm approximately) the assumption of a constant light modulation depth is valid and overestimates the real $m(z)$ more than 50% for thicker samples.

In Fig. 2 we show the variation along sample thickness, neglecting absorption, of the grating amplitude $\kappa_1(z)$ (continuous