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Open boundary conditions in stochastic transport processes with pair-factorized steady states

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Abstract

Using numerical methods we discuss the effects of open boundary conditions on condensation phenomena in the zero-range process (ZRP) and transport processes with pair-factorized steady states (PFSS), an extended model of the ZRP with nearest-neighbor interaction. For the zero-range process we compare to analytical results in the literature with respect to criticality and condensation. For the extended model we find a similar phase structure, but observe supercritical phases with droplet formation for strong boundary drives.

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1. Introduction

Stochastic mass transport processes such as the asymmetric simple exclusion process (ASEP) or the zero-range process (ZRP) proposed by Spitzer (1970) are simple transport models for particle hopping to improve the understanding of basic phenomena in the dynamics of particles in driven diffusive systems. Generally these particles are abstract and may represent objects from the microscopic to the macroscopic scale depending on the situation and their specific dynamics. Just as these particles and their interactions, the underlying spatial structure is an important factor in adapting and mapping these models to physical processes. In this work we will consider two such processes, that feature the formation of particle condensates in periodic systems driven far from equilibrium. We are interested in studying these processes in a situation, where the system is driven by the flux of particles entering and leaving through open boundaries. First we look into the zero-range process, where we can compare our numerical results with analytic predictions available from the literature. Then we discuss the effect of open boundaries on the condensation phenomenon for an extended transport process with short-range interactions as realized by the pair-factorized steady states (PFSS) model introduced by Evans (2006) and Wacław et al. (2009a, 2009b).

2. Zero-range process

The basic stochastic mass transport process of particle hopping consists of a gas of indistinguishable particles on a one-dimensional lattice with L sites. Every site i can be occupied by any number of particles m_i. In the zero-range

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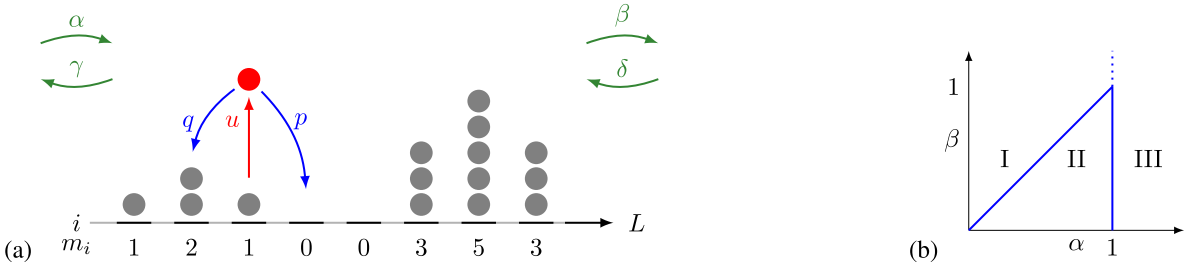

Fig. 1. (a) Schematic representation of the zero-range process on a one-dimensional lattice with L sites and particle injection (rates α, δ) and removal (rates γ, β) through open boundaries at sites $i = 1, L$. (b) Phases induced by boundary drive in the discussed transport processes. A description of the phases is given in the sections of the respective transport process.

process, in every time step of the discrete stochastic time evolution, a random site i is selected, where a single particle may leave to a neighbor with the hopping rate $u(m_i)$. That is, particles only interact with other particles on the same site. The direction of the hop is determined randomly, often with respect to rates that introduce asymmetric dynamics. For an overview of different dynamics we refer, to the book by Schadschneider et al. (2011) or the review by Schütz (2001).

In this work we shall consider the model with hopping rates $u(m) = 1 + b/m$ on a one-dimensional lattice with open boundary conditions also discussed by Evans (2000) and Kafri et al. (2002). Under periodic boundary conditions, the main feature of this model is the formation of a single-site condensate consisting of all particles exceeding a critical density $\rho_c = 1/(b-2)$ for $b > 2$ such that all other sites have an average occupation equal to ρ_c. Effects of open boundaries on this process have been studied analytically by Levine et al. (2005), to which we shall first compare our results before we continue to an extended model. Particle exchange at the boundaries at the first (last) site is achieved by injection with rates α (δ) and removal with rates γ (β), respectively, as illustrated in Fig. 1(a).

In the analytic study by Levine et al. (2005), local fugacities are determined using a quantum Hamiltonian approach proposed in Schütz (2001) to find the phase structure given in Fig. 1(b) and respective properties of the phases with respect to the boundary rates: Phase I, for $\alpha \leq 1, \alpha \leq \beta$, is the only phase, where the system has a steady state. The total number of particles $M = \sum_{i=1}^{L} m_i$ remains stable, the occupation number distribution becomes $P(m_i = m) = \alpha^m/m^b$ and the resulting particle density in the bulk system is subcritical, $\rho_{\text{bulk}} < \rho_c$. In phase II, for $\alpha \leq 1, \alpha > \beta$, the rate of particle influx outweighs that of outflux and particles pile up at the boundary site(s) before leaving, forming one or two condensates in the totally asymmetric ($p = 1, q = 0$) or symmetric ($p = q = 1/2$) cases, respectively. Condensation specific properties in the bulk are not analytically determined. In phase III, for $\alpha > 1$, the rate of particles entering at the boundary site(s) exceeds the maximal possible hopping rate so that particles pile up on the boundary site(s) after entering the system and the total number of particles grows linearly in time.

We will compare only to the cases where the injection and removal rates at each boundary are equal in the symmetric process, $\alpha = \delta$ and $\beta = \gamma$, and $\gamma = \delta = 0$ for the totally asymmetric case. In both cases we use the hopping parameter $b = 5$.

By numerical simulation these phases and properties are easily reproduced and visible in observables such as the total number of particles M over time as shown in Fig. 2(a) with the same phase boundaries as predicted. A more detailed look at the criticality of the system with respect to the boundary drive is possible by computing the bulk density deep inside the system as shown in Fig. 3 for the (a) totally asymmetric and (b) symmetric processes. In the totally asymmetric case, the condensate on the last site in phase II cannot contribute to the bulk density which remains low and subcritical. In phase III, a particle condensate forms at the first site acting as a reservoir for the bulk of the system that becomes critical ($\rho_{\text{bulk}} = \rho_c = 1/(b-2) = 1/3$) as it acts like the bulk in a periodic system with a condensate. In contrast, in the symmetric system in both phases II and III these condensates form at the two boundary sites and particles may hop back into the system, so that the bulk becomes critical already for $\alpha > \beta$ in phase II. Very small droplets of several particles become visible in the bulk, but the monotonic falloff of the distribution of occupation numbers in the phases II and III confirms the absence of condensates in the bulk.