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Ginzburg-Landau calculations of circular $Mo_{80}Ge_{20}$ plates with sector defect

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Abstract

We present the theoretical calculations on vortex structures in a nanosized superconducting circle with a deficit sector. The numerical calculations of Ginzburg–Landau equation has been carried out with the aid of the finite element method, which is convenient to treat an arbitrarily shaped superconductor. We found that the vortices form an arc structure or a partial shell structure in a deficient circle plate, and mirror symmetry can be seen with respect to the sector deficit. Due to the vortex-vortex interaction and the boundary confinement effect, we also found the evolution of double (outer and inner) shell structure as a function of vorticity. Our theoretical studies will be compared to the experimental studies.

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Keywords: Vortex states; Ginzburg-Landau calculations; SQUID microscope

1. Introduction

The distribution of vortices in mesoscopic superconducting disks, of which the size is comparable to the coherence length ξ and the magnetic penetration depth λ, has been explored to discover exotic vortex states. Such features cannot be observed for a bulk superconductor, but it could be caused in the mesoscopic system driven by the vortex–vortex interaction and the boundary–vortex interaction. The formation of different vortex configuration was dependent on geometry of pattern, the external magnetic field, and vorticity [1, 2, 3]. The symmetric vortex structure has been investigated by using the regular polygons with mirror symmetric lines. Several recent studies on mesoscopic disks have clarified the question of how vortices are arranged by the sample geometry. For a small circular disk, the formation of a ring-like structure was arranged under the influence of the sample boundary plays a

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crucial role on vortex distribution [2, 4]. In the case of the noncircular geometry such as an equilateral triangle, [5, 6] or a square [7, 8], the symmetry (C3, C4) of the boundary condition tends to impose the particular vortex arrangement in those system. For instance, with the same vorticity L, the vortex configuration is different in square and triangle plates [9]. Beside the standard pentagon, several studies introduced an artificial element such as a pinning center to make the vortex configuration asymmetric [1] which was used to observed the vortex interaction, or changing sides of pentagon to become star to make complex structure [10] which observed a distinction configuration of vortex in concave decagon. Both theoretical and experimental approaches have used to explore the distribution of vortices, i.e., the nonlinear Ginzburg–Landau theory and the London approximation were used [11, 12, 13] while scanning SQUID microscopy was applied to small superconducting $Mo_{80}Ge_{20}$ plates [3, 5].

In this paper, we present theoretical prediction about the distribution of vortex in a circle disk with sector defect (Pakman [14]). In this pattern, there are several features. (1) The super current is inflected when it follows along the sample edge of Pakman shapes. At the deep corner of the deficit sector, the magnetic field is weakened, and hence the vortices tend to penetrate into the interior of the plate, in which we consider that the sector-shaped defect acts as a gate way for allowing the intentional entrance of vortices into the circle disk. It is our interest to see not only the symmetry in vortex configuration but also the influence of the open angle of the deficit sector upon the shell structure of vortices. We consider that the number of vortices to form shell structure was affected by the angle of each sector defect because of the limited length of the partial shell (arc) length. The vortex-vortex interaction and the supercurrent confinement would lead to the formation of the two-shell structure. We found that the number of vortices at an outer shell was dependent on the number of inner-shell vortices when we increased the magnetic field systematically. We investigated the vortex profiles in the Pakman-shaped plate both theoretically and experimentally. Although we found a reasonable agreement between the theoretical predictions and the experimental findings by using a scanning SQUID microscope on the small Pakman plate, we would like to explain the theoretical results on vortex configuration in the present paper.

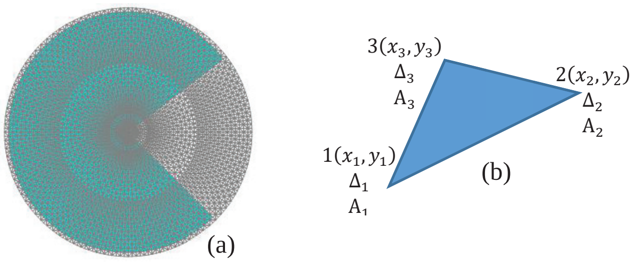

Fig.1.(a) Division of a Pakman superconductor into a number of triangular elements; (b) a triangular element. At each node, values of an order parameter and a magnetic vector potential are defined.

2. Theoretical formalism

We used the Ginzburg-Landau (GL) calculation in our preceding studies [1, 12]. We rebuilt order parameter and structure of circular plates with sector defect by means of finite element method. We consider a Pakman-shaped plate consisting of two-dimensional nanosized superconducting film, to which the external magnetic field H was applied perpendicular. In order to obtain stable vortex structures, the GL free energy is written in terms of a complex order parameter as:

$$\mathcal{F}(\Delta, A) = \int d\Omega \left[\frac{1}{2}\left(\sqrt{\beta}|\Delta|^2 + \frac{\alpha}{\sqrt{\beta}}\right)^2 + \frac{1}{4m}\left|\left(\frac{\hbar}{i}\nabla + \frac{2e}{c}A\right)\Delta\right|^2 + \frac{|\nabla\times A - H|^2}{8\pi} + \frac{1}{8\pi}(divA)^2 \right] \tag{1}$$

where $\Delta(r)$ is an order parameter of a superconductor, A is the magnetic vector potential and H is the external magnetic field. The parameter β and α are a positive constant, α depends on temperature as $\alpha = \alpha(0)(1- T/T_c)$. In this study, we consider that the temperature of the system is well below the transition temperature T_c, and hence is assumed to be positive.

$$\Delta(x, y) = N_1(x, y)\Delta_1 + N_2(x, y)\Delta_2 + N_3(x, y)\Delta_3 \tag{2}$$

$$\boldsymbol{A}(x, y) = N_1(x, y)\boldsymbol{A}_1 + N_2(x, y)\boldsymbol{A}_2 + N_3(x, y)\boldsymbol{A}_3 \tag{3}$$

where $N_i(x, y) = (a_i + b_i x + c_i y)/2S_e (i = 1,2,3)$ are area coordinates, and Δ_i and $\mathbf{A}_i$ are the values of the order parameter and the magnetic vector potential at i-th node, respectively (see Fig. 1b). $a_i = x_i y_k$, $b_i = y_i - y_k$ and $C_i = x_k - y_j$ are defined using coordinates of nodes $(x_i, y_i z_k)$ of the triangular element and S_e is an area of the element. Using this expansion for minimizing the free energy, we obtain following equations for the order parameter,

$$\textstyle\sum_j \left[P_{ij}(\{A\}) + P_{ij}^{2R}(\{\Delta\})\right] Re\Delta_j + \left[Q_{ij}(\{A\}) + Q_{ij}^{2}(\{\Delta\})\right] Im\Delta_j = V_i^R(\{\Delta\}) \tag{4}$$

$$\textstyle\sum_j \left[-Q_{ij}(\{A\}) + Q_{ij}^{2}(\{\Delta\})\right] Re\Delta_j + \left[P_{ij}(\{A\}) + P_{ij}^{2I}(\{\Delta\})\right] Im\Delta_j = V_i^I(\{\Delta\}) \tag{5}$$

where the coefficients are defined as

$$P_{ij}(\{A\}) = \textstyle\sum_{\alpha=x,y} K_{ij}^{\alpha\alpha} + \sum_{\alpha=x,y} \sum_{i_1 i_2} I_{i_1 i_1 i j} A_{i_1\alpha} A_{i_2\alpha} - \frac{A_{ij}}{\xi^2} \tag{6}$$

$$P_{ij}^{2\binom{R}{I}}(\{A\}) = \frac{1}{\xi^2} \textstyle\sum_{i_1 i_2} I_{i_1 i_2 i j} \left(\binom{3}{1} Re\Delta_{i_1} Re\Delta_{i_2} + \binom{1}{3} Im\Delta_{i_1} Im\Delta_{i_2} \right) \tag{7}$$

$$Q_{ij}(\{A\}) = \frac{2}{\xi^2} \textstyle\sum_{\alpha=x,y} \sum_{i_1} (J_{j i_1 i}^{\alpha} - J_{i i_1 J}^{\alpha}) A_{i_1}^{\alpha} \tag{8}$$