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Review

Collective learning modeling based on the kinetic theory of active particles

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Abstract

This paper proposes a systems approach to the theory of perception and learning in populations composed of many living entities. Starting from a phenomenological description of these processes, a mathematical structure is derived which is deemed to incorporate their complexity features. The modeling is based on a generalization of kinetic theory methods where interactions are described by theoretical tools of game theory. As an application, the proposed approach is used to model the learning processes that take place in a classroom.

Keywords: Kinetic theory; Active particles; Stochastic differential games; Social learning; Monte Carlo particle method

1. From five key questions to the plan of the paper

Since the first half of the last century, some pioneering studies were devoted to the mathematical formulation and representation of learning phenomena [54–56,61,74], followed over the years by a growing interest toward the derivation of suitable theories and models. In particular, [29] is a survey of several studies developed in the '50s where two main schools of thinking are pointed out. The models related to the first one are known in the literature as "stimulus sampling models". The notion of a stimulus situation was first formalized in a theory developed to account for the effects of repetition in the acquisition process. Natural extensions of this theory have led to interpretations of discrimination, generalization, temporal processes and even motivational phenomena. In [43], it is assumed that the subject in a learning experiment samples a population of stimuli, or "cues", on each trial and that his probability of making a given response depends on the proportion of sampled stimuli that are "conditioned" or "connected" to the response. Moreover the connections between stimuli and response may be reinforced or weakened during the learning process.

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The other main line of research was concerned with the time sequences of responses of different trials. Such research is referred to as "stochastic learning models". These models were designed to describe the responses made by subjects in simple repetitive experiments [30]. The subject of the experiment receives a stimulus, then he makes one of a number of possible responses and, accordingly, receives either a reward or a punishment. It is assumed that the responses have a probability of occurrence which evolve in time depending on the outcome of each trial. The learning process consists of the changing probabilities of the responses and the rules that modify them. It is worth noticing that, as pointed out in [29], the above mentioned two main research lines are not mutually exclusive but are instead complementary. In more recent years several studies were dedicated to the formulations of mathematical algorithms which could approximate the "knowledge" of some unknown information. In this respect, in [37] the authors present an interesting survey which characterizes the mathematical theory of learning as a method to approximate systems described by a large number of variables. Among the other examples reported in [37], it is worth pointing out the learning of a physical law by curve fitting to data and the pattern recognition of objects, such as handwritten letters of the alphabets or pictures of animals. Moreover, the above mentioned paper is also recognized for its contribution to compression and interpretation of big dispersed data, which nowadays demand an increasing research activity to mathematicians and statisticians.

According to [37], the mathematical modeling is essentially related to two fundamental types of learning, namely the cognitive learning and the social learning. The *cognitive learning* refers to the case when an individual increases its mental knowledge. In this category we find the "latent" learning which occurs when the acquired knowledge is manifested with some delay and the learning "by insight", a theory first put forth by Kohler [67], which is the abrupt realization of the problem solution. The *social learning* instead, occurs when the individual learns new behaviors from others. In this category we find the learning by "imitation" [11], where "the others" constitute the model of behavior and in turn the model acts as a confirmation. To the same category belongs the learning by "tradition", good examples of which are the language acquisition by children and behavioral learning in the animal kingdom. In such context it is also worth stressing that one of the great contributions to biology is the replacement of the concept of *typological thinking* by that of *population thinking*, proposed by Mayr [71]. The new idea of population thinking linked to the concept of mutations and selection can explain various aspects of the theory of evolution. Mayr's studies strongly motivated in later years the development of the evolutionary game theory [59]. These fundamental scientific contributions have an important impact also on the contents of this paper which cares about the formalization by mathematical structures of the aforementioned concepts.

After [37], a vast literature developed on the aforementioned two types of learning, also due to the growing interest toward living, hence complex, systems composed of many interacting agents [9]. Examples include crowds, swarms and schools [22], intelligent robots [24,32,62,69] and social systems in general [5,58,82]. In particular, as pointed out in [45], even a simple cooperative strategy based on imitative learning, allows a group of agents to work efficiently as a connected collective brain. Indeed, through cooperation, the agents obtain a better performance than working in isolation.

Learning dynamics can have an important role in social dynamics [52], where interactions contribute to diffusion and modifications of opinions [25,26,57], thus leading to political choices including support or opposition to governments [19]. An important reference is offered by the stochastic evolutionary game theory [7,31,76], to be properly cast into a differential framework.

This paper, based on the aforementioned motivations and on some perspective ideas proposed in [39], pursues the objective of developing a mathematical theory of perception and learning in view of their application to modeling complex systems, which can develop a collective intelligence [12,27,35]. We believe that our formulation can be viewed as an extension of the concept of population thinking and of the theory of evolution reported in [71,72]. Indeed, this aspect will motivate us to relay on the tools of the evolutionary game theory.

The mathematical tools are derived from the methods of the classical kinetic theory [33], statistical dynamics [58] stochastic evolutionary game theory [7,50,76] and their development toward the theory of active particles [20]. The latter was specifically developed to model living systems constituted of several multi-agents interacting by linear or nonlinear rules. Over the years this approach has been applied in a variety of different fields such as spread of epidemics [40], social systems [6], micro-scale Darwinian evolution and selection [23] and collective learning process [34]. It is worth stressing that important motivations to the contents of this paper are induced by the idea that the mathematical structure might include features which could make it interesting in different field of life sciences [53].