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Reply to comment

Learning dynamics towards modeling living systems Reply to comments on “Collective learning modeling based on the kinetic theory of active particles”

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1. Introduction

Our paper [19] presents a review and critical analysis on a mathematical theory of learning in populations composed of many interacting individuals. Furthermore, it attempts to provide a foundational mathematical framework which may incorporate the main features of the learning process in view of applications to modeling complex systems, including crowds [39,15], swarms [2,4], and social systems [1,24,35,41].

The proposed approach is based on the kinetic theory of active particles which has been specifically developed to deal with living systems [17]. The novelty of the contribution is the focus on collective, rather than individual, learning dynamics. This topic presents a certain analogy with evolutionary game theory, where populations take the place of individual players [36].

We wish to thank all commentators for sharing their insightful and stimulating ideas on the topic [5,10,22,23,37,38,40,45,46]. Their comments provide a variety of perspectives on the collective learning which have helped to further refine our thoughts, and to broaden the scope of our review [19]. Indeed, we consider our paper [19] only a first step towards this challenging research topic which will definitely be followed by further developments.

In the next two sections we review the comments and provide an outlook of the enlightened research perspectives.

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2. Analysis of the comments

We here summarize and analyze the contents of the comments with the aim of understanding how the hints posed by them can be put in practice by an appropriate research activity. According to their topic, comments may be grouped into four categories.

Collective learning and life sciences: The role that the learning process can play on the dynamics of systems in various fields of life sciences is an important topic raised by the most of the comments. More specifically, the comments by Dolfin [23], Knopoff [37], and Urrutia [46] refer to recent papers in the field of social sciences [16,24,25] while the comment by Bellomo [10] focuses on biology [14]. We think that these comments are extremely well focused. Indeed, although collective learning is an ubiquitous process in social and biological systems, the greatest variety of models presented in the literature dealt with it only in a simplified manner. On the other hand, a more realistic modeling can lead to new results as discussed in the last part of this reply.

Collective learning and space heterogeneity: In spite of its importance, the collective learning in a population of individuals heterogeneously distributed in space is only partially treated in the literature. This topic is posed in the comments by Chouhad [22] and Nieto [40]. More in detail, the first one refers to applications to systems where interactions occur continuously in space, such as vehicular traffic and human crowd [15,26] while the second one enlightens the role of space in networks and provides hints for modeling the perception domain. The development of stochastic game theory for particles interacting in space is a challenging research perspective which is certainly worth to be investigated.

Collective learning and scaling problem: The various aspects of perception and learning can be described at different scales, namely, microscopic, mesoscopic, and macroscopic. This topic has been addressed in the comments by Chouhad [22] and Lachowicz [38]. In particular, these comments focus on the challenging analytic problem related to the derivation of macroscopic equations from the underlying description at the microscopic scale. This issue is of great importance for both theoretical interest and practical relevance. We stress that the scaling problem plays an important role in the modeling approach. Specifically, the hyperbolic scaling [12] leads to equations that model phenomena with finite speed which appear to be consistent with a large variety of propagation phenomena of real systems.

Collective learning and classical kinetic theory: The comments by Banasiak [5] and Shizgal [45] focus on the connection between classical kinetic theory, in particular the Boltzmann equation, and the approach proposed in our paper. These interesting comments give us the opportunity of making more precise the conceptual differences. The analogy is due to the dependent variable, which in both cases is a probability distribution over the particle microscopic state. The fact that in the Boltzmann equation such a state is a velocity, while in our case is an activity variable, is a simply technical matter. On the other hand, a substantial difference is related to interactions which in the former model are described by mechanics, while in the latter by social exchanges modeled by game theory. This feature has allowed us to include the dynamics over networks as also observed in [5] referring to well defined applications [6,7]. This conceptual difference induces the need of developing computational methods, as observed in [45] and discussed in the next section.

3. Research perspectives

We have been pleased to notice that the authors of all comments share the main idea of our paper that collective learning is an important process which significantly influences the dynamics of systems composed of many interacting living entities. We here take the liberty of transferring some of their stimulating hints into preliminary thoughts looking ahead to future research on the subject. It is worth stressing that the challenging research activity outlined in the following may take advantage from tools and concepts of different disciplines, such as neural [27,28] and cognitive [42,43] sciences, and needs a deep understanding of the physics of the system under consideration.

- *Mathematical problems:* Comments [22] and [38] have brought to our attention two mathematical topics which, according to our own bias, are worth to be investigated, namely the development of a game theory for systems interacting in space and the derivation of macroscopic models from the underlying description at the microscopic scale. Both topics have already been treated in the literature, the former is reviewed in [17], the latter in a sequel of papers including the reference to models of vehicular traffic [13] and crowd dynamics [11], already cited