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Review

# Thermostatted kinetic equations as models for complex systems in physics and life sciences ☆

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**Abstract**

Statistical mechanics is a powerful method for understanding equilibrium thermodynamics. An equivalent theoretical framework for nonequilibrium systems has remained elusive. The thermodynamic forces driving the system away from equilibrium introduce energy that must be dissipated if nonequilibrium steady states are to be obtained. Historically, further terms were introduced, collectively called a thermostat, whose original application was to generate constant-temperature equilibrium ensembles. This review surveys kinetic models coupled with time-reversible deterministic thermostats for the modeling of large systems composed both by inert matter particles and living entities. The introduction of deterministic thermostats allows to model the onset of nonequilibrium stationary states that are typical of most real-world complex systems. The first part of the paper is focused on a general presentation of the main physical and mathematical definitions and tools: nonequilibrium phenomena, Gauss least constraint principle and Gaussian thermostats. The second part provides a review of a variety of thermostatted mathematical models in physics and life sciences, including Kac, Boltzmann, Jager–Segel and the thermostatted (continuous and discrete) kinetic for active particles models. Applications refer to semiconductor devices, nanosciences, biological phenomena, vehicular traffic, social and economics systems, crowds and swarms dynamics.


*Keywords:* Complexity; Thermostats; Dynamical systems; Nonlinear ODE and PDE; Kinetic theory

**Contents**



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## 1. Introduction and motivations of the review

Complex phenomena arising in physics and life sciences systems emerge from interactions occurring in nonlinear fashion among many elements of the system that as a whole exhibits one or more properties (*emerging behaviors*) not obvious from the properties of the individual parts [12]. Mathematicians and physicists have long hoped that these collective behaviors could be described using the ideas and methods of statistical mechanics. Indeed different approaches inspired to equilibrium or nonequilibrium statistical mechanics have been developed, adapted and employed in an attempt to describe collective behaviors and macroscopic features as the result of microscopic (individual) interactions, [37,101,124,145,160,194,217].

Complex behaviors appearing when we deal with the inert matter are very different by complex phenomena appearing in living systems. Indeed the emerging behaviors are also consequence of the ability of individuals to develop specific and autonomous strategies. As a matter of fact, what it is found especially interesting about the behavior of multicellular organisms emerges from interactions among many cells, and the most striking behaviors of animal populations are similarly collective. Nevertheless the main difficulty in living systems is that they are not really equilibrium statistical mechanics problems, therefore there is no guarantee that we can find relevant macroscopic variables, e.g. the spin glass is the correct description of a neural network, see papers [193,206,208] and the references therein, but it is not clear how to measure the analog of the magnetic susceptibility. Moreover, what is really hard in complex living systems is the existence and modeling of asymptotic nonequilibrium stationary states.

The complexity in the stationary states of living systems relies on their structure that is not well ordered as a crystal but it is not chaotic and disordered like a gas. Further, a constant flow of energy and material through the system maintains these states far from equilibrium. Statistical mechanics has suggested to characterize these states by means of the theory of self-organized criticality that has been originally applied for the modeling of inert matter systems but recently has been proposed also for living systems, see [7,120,161,162,165,168,181,185,191].

Critical phenomena are typical of equilibrium systems, but there are very few cases where equilibrium properties are relevant to life. Criticality, however, is a much more general concept than its instantiation by phase transitions in equilibrium systems. The description of statistically stationary states of living systems can be performed by a probabilistic approach: *the probability of finding the system in a particular state is governed by a probability distribution that is mathematically equivalent to the Boltzmann distribution for a system poised at a critical point*.

Usually in critical phenomena there are some natural macroscopic variables with a singular dependence on parameters that can be controlled experimentally, e.g. the critical point of a gas can be identified by measuring the density of the fluid as a function of temperature and pressure. Criticality can be also identified in purely thermodynamic measurements, i.e. looking the behavior of the correlation function of fluctuations in some local variable.

Experimental evidence of criticality have been observed in a wide variety of complex living systems, especially biological systems, spanning all possible scales, from individual proteins to whole populations of animals with high cognitive capacity, such as schooling fish, swarming insects or flocking birds [155]. Indeed seems that the collective