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# Generation of impulses from single frequency inputs using non-linear propagation in spherical chains

D. Hutchins[a], J. Yang[a], P. Thomas[a], O. Akanji[a], L. Davis[a], P. Gelat[b], N. Saffari[b], S. Harput[c] and S. Freear[c]

[a]*University of Warwick, School of Engineering, CV4 7AL Coventry, UK*
[b]*University College London, Dept. of Mechanical Engineering, WC1E 7JE London, UK*
[c]*University of Leeds, School of Electronic and Electrical Engineering, LS2 9JT Leeds, UK*

**Abstract**

This paper investigates the use of chains of spheres to produce impulses. An ultrasonic horn is used to generate high amplitude sinusoidal signals. These are then input into chains of spheres, held together using a minimal force. The result is a non-linear, dispersive system, within which solitary waves can exist. The authors have discovered that resonances can be created, caused by the multiple reflection of solitary waves within the chain. The multiply-reflecting impulses can have a wide bandwidth, due to the inherent nonlinearity of the contact between spheres. It is found that the effect only occurs for certain numbers of spheres in the chain for a given input frequency, a result of the creation of a nonlinear normal mode of resonance.The resulting impulses have many applications, potentially creating high amplitude impulses with adjustable properties, depending on both the nature and number of spheres in the chain, and the frequency and amplitude of excitation.


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## 1. Introduction

There have been various studies that have investigated acoustic propagation along chains of spheres, where acoustic propagation is non-linear due to Hertzian contact between each sphere Coste C. et al (1997). This causes interesting features, one of which is the creation of solirtary waves, provided certain conditions are met. Nesterenko (1983) has described how solitary waves can be established in long chains of spheres when the contact force holding them together is extremely small. Spadoni and Daraio (2010) demonstrated that solitary waves in chains exist when steel ball-bearings are held within columns and a small pre-compression ($F_0$) applied, with a later study extending


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the analysis using piezoelectric actuation (Donahue et al (2014)). The resultant effects are dependent on the characteristics of the applied transient force ($F_m$), the diameter of the spheres, and the relative values of $F_m$ and $F_0$. If $F_m \approx F_0$, weakly non-linear behavior results, whereas if $F_m >> F_0$, propagation along the chain is highly non-linear.

Most of the experimental work to date has used relatively low ultrasonic frequencies and large spheres. The cut-off frequency for solitary waves limits the upper frequency that can propagate in chains; hence, higher frequencies will need smaller spheres. In addition, little work seem to have been performed experimentally into establishing whether higher frequencies and non-impulsive waveforms could be used in finite lengths of chain. Of particular interest was the establishment of resonances within such chains. These are likely to be complicated, due to non-linear propagation from one sphere to the next. Various papers have described the concept of Non-linear Normal Modes (NNMs) of vibration, which are expected to exist in such systems (Jayaprakash et al (2011); Starosvetsky and Vakakis (2009); Lydon et al (2013)). NNMs are expected to be very sensitive to the input conditions, in terms of the nature of $F_m$ and $F_0$, the types of spheres used, and the boundary conditions.

The present paper describes a series of experiments that have investigated the behavior of spherical chains of variable length, and where the ratio of $F_m/F_0$ could be varied. To achieve this at ultrasonic frequencies, it was necessary to have a high amplitude ultrasonic input. This was supplied from an ultrasonic horn at a fixed resonant frequency. As will be shown, this leads to the generation of very interesting phenomena, and in some cases, to the creation of a set of impulses (Hutchins et al, 2015).

## 2. Apparatus and Experiment

The apparatus was designed to allow signals from an ultrasonic horn at 73 kHz to be transmitted into one end of a chain of 1 mm diameter stainless steel spheres, held within a cylindrical holder, as shown in Fig. 1. The last sphere was held in position via an annuluar end-plate, so that a vibrometer could be used to measure the acoustic velocity waveform at the far end of the chain. A static force $F_0$ could be applied at known values to this end plate. The signal supplied by the ultrasonic horn was also adjustable in terms of both amplitude (in terms of acoustic velocity ampltitude $v_m$) and the number of cycles in the waveform. Ouput signals were recorded for later analysis using a digital oscilloscope.

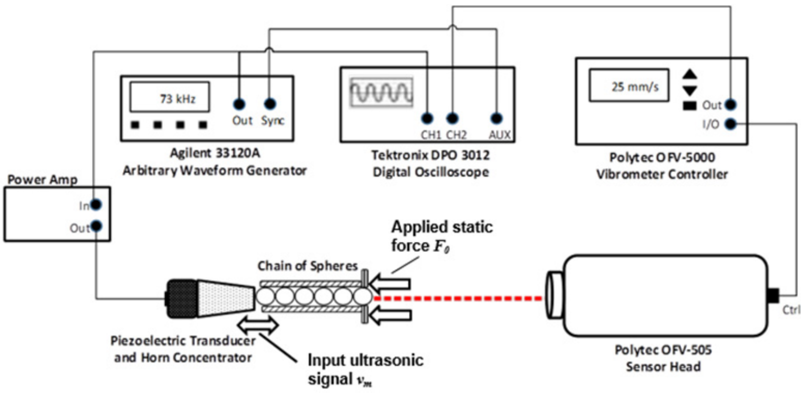


Fig. 1. Schematic diagram of apparatus.

## 3. Theory

The experimental results could be compared to theory, using solutions to the dynamic equations for spheres in a chain. Details are given elsewhere (Hutchins et al, 2015), but a point to note is that the results are highly dependant on the boundary conditions – the contact of the first sphere to the horn, and of the last sphere to the annular end-plate of the holder, must both be modelled properly. The theory assumes that the centres of each sphere move