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Fatigue life prediction using multiaxial energy calculations with the mean stress effect to predict failure of linear and nonlinear elastic solids

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ABSTRACT

An approach is presented that enables the calculation of elastic strain energy in linear and nonlinear elastic solids during arbitrary thermomechanical load cycles. The approach uses the simple fact that the variation of both strain and complementary energies always forms a rectangular shape in stress–strain space, hence integration is no longer required to calculate the energy. Furthermore, the approach considers the mean stress effect so that predictions of fatigue damage are more realistically representative of real-life experimental observations. By doing so, a parameter has been proposed to adjust the mean stress effect. This parameter α is based on the well-known Smith–Watson–Topper energy criterion, but allows consideration of other arbitrary mean stress effects, e.g. the Bergmann type criterion.

The approach has then been incorporated into a numerical method which can be applied to uniaxial and multiaxial, proportional and non-proportional loadings to predict fatigue damage. The end result of the method is the cyclic evolution of accumulated damage. Numerical examples show how the method presented in this paper could be applied to a nonlinear elastic material.

Introduction

Mechanical components are usually subjected to variable loads during operation. These cause stresses, strains and temperature rises in the component as a reaction to such loading. Depending on the size of the load and the exposure (operating) time, fluctuating stress–strain fields in a component can eventually lead to a crack where the damage is greatest – the critical location, which will continue to grow under continued loading and eventually result in the failure of the component [1–5]. Although various mechanisms lead to the deterioration of mechanical products once they are put into operation, fatigue is still one of the main sources of failure for products that operate over longer amounts of time, e.g. months, years or hundreds of thousands of load cycles [6,7]. Fatigue mechanisms prosper due to the changeable load and environmental conditions and can ultimately lead to a complete stoppage of functionality of these products [7–11].

Predicting fatigue damage of a product is therefore not only important directly prior to manufacture but is an integral step in the early stages of product development. However, the identification of critical locations and the quality of the prediction, e.g. the predicted number of cycles to failure, will only be as good as the following: level of complexity of the temperature dependent stress–strain calculation; reproducibility of the damage accumulation modelling; level of detail included in the fatigue damage prediction; and accuracy of the input data of the material properties [9,11]. The final experimental verification of the prediction is always valuable before the component enters the manufacturing stage but prior to this stage, computer aided predictions are necessary as a means of reducing financial outlay and shortening development times [6,12].

The majority of fatigue damage predictions are still based on uniaxial approaches or transformations of multiaxial stress–strain states into equivalent (uniaxial) cases either assuming a failure theory (e.g. signed von Mises stress) or applying the critical plane approach [5,12–17]. They usually give satisfactory predictions, especially if they incorporate various influences on the fatigue damage prediction such as e.g. the mean stress correction [5,17–21]. However, under more complex conditions some of the commonly used techniques may no longer be capable of producing accurate predictions [16,17,19,21]. Alternatively, the invariance of the energy (which is independent of the coordinate system of observation) and its dissipation during cyclic loading have proven to be a suitable tool for predicting fatigue damage regardless of the type of loading (mechanical, thermal, uniaxial, multiaxial proportional or non-proportional) [13,18,22,23]. Therefore energy-based models for fatigue damage predictions have been a good counterweight to equivalent prediction models. However, according to the available literature, there have been attempts to include the mean

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Nomenclature

α	mean stress parameter
δ	Prandtl density
$\mathcal{D}$	damage operator
$\Delta\varepsilon_{a,p}$	elastic principal strain range
$\Delta\varepsilon_{ae,p}$	equivalent elastic principal strain range
$\Delta U_{ae,p}$	total equivalent elastic principal energy range
ε_a^{cycle}	uniaxial experimental strain amplitude
$\varepsilon_{a,p}$	elastic principal strain amplitude
$\varepsilon_{ij}, \varepsilon_{kl}$	elastic strain
$\varepsilon_p, \varepsilon_q$	elastic principal strain
$\varepsilon_{j,p}^{res}$	residuum principal strain
$\varepsilon_{a,p}$	elastic principal strain amplitude
$\varepsilon_{ae,p}^*$	equivalent linear elastic principal strain amplitude
$\varepsilon_{ae,p}$	equivalent nonlinear elastic principal strain amplitude
$\varepsilon_{max,p}$	maximum absolute principal strain
$\varepsilon_{o,p}$	origin of elastic principal strain
σ_a^{cycle}	uniaxial experimental stress amplitude
$\sigma_{a,p}$	principal stress amplitude
$\sigma_{ae,p}^*$	equivalent linear principal stress amplitude
$\sigma_{ae,p}$	equivalent nonlinear principal stress amplitude
σ_{ij}	stress
σ_m^{cycle}	uniaxial experimental mean stress
$\sigma_{m,p}$	principal mean stress
$\sigma_{max,p}$	maximum principal stress
$\sigma_{o,p}$	origin of principal stress
σ_p	principal stress
C	elastic complementary energy per unit volume
C_a	elastic complementary energy amplitude
$C_{a,p}$	elastic principal complementary energy amplitude
$C_{ae,p}$	equivalent elastic principal complementary energy amplitude
C_m	elastic mean complementary energy
$C_{m,p}$	elastic principal mean complementary energy
d, d_f	equivalent cycle damage
D, D_f	accumulated fatigue damage
D_a	damage due to total elastic energy amplitude
D_{ae}	damage due to total equivalent elastic energy amplitude
D_m	damage due to total elastic mean energy
D_{ijkl}	nonlinear elastic stiffness
D_{ppqq}	linear elastic stiffness
i, j, k, l	second and fourth order tensor indices
i	stress history index (only in Appendix B)
j	index for number of reversal points in residuum (only in Appendix B)
j	stress index of the spring-slider model
k	temperature index of the spring-slider model
n_s	number of time increments
n_u	number of fictive yield energies
n_T	number of temperature divisions
p	index of principal components
R_ε	strain ratio
R_σ	stress ratio
s	time index
S	logical operator (only in Appendix B)
t	time
T	temperature
u	yield surface
U_a	total elastic energy amplitude
$U_{a,p}$	total elastic principal energy amplitude
U_{ae}	total equivalent elastic energy amplitude
U_{ae}^{cycle}	total experimental elastic energy amplitude
$U_{ae,p}$	total equivalent elastic principal energy amplitude
$U_{\delta j}$	back stress of the spring-slider model
$U_{e,p}$	total equivalent elastic principal energy
U_e	total equivalent elastic energy
U_m	total elastic mean energy
$U_{m,p}$	total elastic principal mean energy
$U_{o,p}$	origin of total equivalent elastic principal energy
W	elastic strain energy per unit volume
W_a	elastic strain energy amplitude
$W_{a,p}$	elastic principal strain energy amplitude
W_{ae}	equivalent elastic strain energy amplitude
$W_{ae,p}$	equivalent elastic principal strain energy amplitude
W_m	elastic mean strain energy
$W_{m,p}$	elastic principal mean strain energy

stress correction in the energy-based methods e.g. [18,22], but to date no established criterion has been accepted as e.g. are the Smith–Watson–Topper (SWT) or Bergmann mean stress criteria for the uniaxial stress–strain states [5,16,18,24,25].

Here we present how energy-based fatigue damage predictions can be applied to a given variable multiaxial thermomechanical loading and nonlinear elastic solid, and hence show how they could be applied to materials such as metals, rubbers, polymer networks, liquid crystal elastomers and new biological materials under large strains. Furthermore, the approach is extended to consider the mean energy influence of the load cycles which can exactly reproduce the well-known uniaxial SWT correction [24–26] or can be adapted for another experimentally observed influence on the mean stress level, i.e. a Bergmann type correction [25] by introducing an additional parameter α. The approach presented here is incorporated into a robust method based on Prandtl operators [6,10,12,23] that estimates the accumulated thermomechanical fatigue damage at any time instant during the load history by calculating the cyclic fatigue damage evolution.

Energy calculation

The material response under cyclic loading is assumed to be temperature dependent and nonlinear elastic. This means that it is independent of the load history (path independent) and that the stress tensor σ_{ij} and strain tensor ε_{kl} form a nonlinear constitutive law

$$\sigma_{ij}(t_s) = \sigma_{ij}(D_{ijkl}(T_s), \varepsilon_{kl}(t_s)) \tag{1}$$

which depends on a temperature dependent stiffness tensor D_{ijkl} and temperature $T_s = T(t_s)$ for every time instant $t_s; s = 1, \ldots, n_s$. It will be assumed here that both stress and strain tensors σ_{ij} and ε_{kl} for every time instant have been determined in advance according to a nonlinear elastic model as the material response modelling is not the main focus of the paper. As the approach is general, the stress and strain tensor can be considered as multiaxial and nonproportional. In the equations below, the time and temperature dependence will be omitted for simplicity though they are considered throughout the calculation. Additionally, all the quantities in this paper apply only to elastic materials, e.g., $\varepsilon_{kl} = \varepsilon_{kl}^{el}, U_e = U_e^{el}$, hence "el" superscripts will be omitted for clarity.

First, strain energy and its complementary energy must be defined. These phenomena are crucial for the calculations to follow. Eqs. 1–13 refer to [27] where the reader can find further details on the strain and complementary energies.

For a given stress state σ_{ij}, an infinitesimal amount of strain energy per unit volume dW (referred to as strain energy hereafter) during a load cycle can be calculated as