Results in Physics 6 (2016) 407–413

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.journals.elsevier.com/results-in-physics

Entanglement, nonclassical properties and geometric phase of Raman photon pairs in the presence of time-dependent coupling

Sadah Alkhateeb [a], S. Abdel-Khalek [b], K. Berrada [c,*], Eman M.A. Hilal [a]

[a] Mathematics Department, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia
[b] Mathematics Department, Faculty of Science, Taif University, Taif, Saudi Arabia
[c] Al Imam Mohammad Ibn Saud Islamic University (IMSIU), College of Science, Department of Physics, Riyadh, Saudi Arabia

ARTICLE INFO

Article history:
Received 25 June 2016
Accepted 14 July 2016
Available online 21 July 2016

Keywords:
Raman photon pair
Time-dependent coupling
Entanglement
Statistical properties
Geometric phase

ABSTRACT

In this paper, we develop the model of the four-level double Raman pairs by exploiting the required optimal conditions for this system that are feasible with real experimental realization. We investigate qualitatively the entanglement, statistical properties, and geometric phase for the pair of Stokes and anti-Stokes photons in the presence of the time-dependent coupling effect. We show that these quantifiers are very sensitive to the change of the Rabi frequency and time, exhibiting substantial phenomena that are depending on the kind of coupling between the atom and photons. Finally, we explore the relationship between the quantum quantifiers in terms of the physical parameters with and without time-dependent coupling effect.

Introduction

Quantum technology science has emerged as a new rich field due to its potential applications [1,2], combining and drawing on the disciplines of physical science, mathematics, computer science, and engineering. Its aim is to study how these tasks can be accomplished and show that the physical laws that are used early can be explored to develop the different tasks of optics and information using the features of the quantum mechanical systems. These quantum characters lead to some advantages and can be exploited to perform new promising tasks which are impossible in the classical realm. A generation of well-controlled non-linearity and non-classicality character is the main research of many scientists working in the relatively new field of quantum technologies. Interactions of matter with light are widely considered in the literature and appeared in most processes of quantum optics and information [3–5]. The study involving light-matter interactions become very popular among scientists theoretically as well as experimentally due to its advantageous application for future technologies. The simple description of quantum light-matter interaction, namely the interaction between a qubit and harmonic oscillator is given by the Rabi model [6].

A generation of well-controlled quantum correlations is the main research of many scientists working in the relatively new field of quantum technologies [1]. One of the studies of quantum correlations is entanglement [7–12]. A prototype system, which can display quantum entanglement, is the bipartite quantum systems. One of the most interesting aspects is the entanglement between light and matter [13,14]. In recent years different devices have been proposed and realized experimentally to generate quantum entanglement, such as beam splitter [15,16], cavity QED [17], and NMR systems [18]. Entangled photon pairs are an integral asset to quantum communication technology with continuous variables [19]. A bright source of entangled photon pairs could be useful also for quantum lithography [20]. The transient entanglement of many photon pairs has been shown to exist for a cascade scheme [21,22], and double Raman scheme [23]. The transient regime does not provide a continuous source of entangled photon pairs that could be as typical lasers in continuous wave operation.

Geometric phase is an example of intrinsic feature in quantum mechanics which has been investigated by almost two generations of physicists. A considerable understanding of the formal description of quantum mechanics has been achieved after Berry's discovery [24–27] of a geometric feature related to the dynamics of a quantum system in the adiabatic and the cyclic unitary evolution of non-degenerate states. There are plenty of generalizations, including non-adiabatic [28], non-cyclic and even non-unitary evolution of quantum states. Berry's showed that the wave function of a quantum object retains the memory of its evolution in its complex phase argument, which apart from the usual dynamical contribution, only depends on the geometry of the path traversed by the system, known as the geometric phase factor. This contribution

* Corresponding author.

http://dx.doi.org/10.1016/j.rinp.2016.07.004
2211-3797/

originates from the very heart of quantum mechanics. The geometric phase can be expressed as a path integral and via the Stokes theorem, and can be converted into a surface integral. Therefore, it behaves like a geometric area. A quantity like an area is less dependent on the details of time evolution and therefore is less affected by changes of environmental conditions or an imperfect control, and hence, is typically more robust. This is the key attribute that makes geometric phase attractive for the implementation of fault-tolerant quantum computation.

Nonclassical photon pairs driven by laser pulses could be made sufficiently intense and serves as a new tool for doing quantum nonlinear optics. Entangled photon pairs are integral assets to quantum communication technology with continuous variables [29]. This represents a new kind of correlated photon state, which is of interest in itself and is likely to find applications in various fields, for example, in stellar astronomy [30], foundations of quantum mechanics [31], subnatural spectroscopy [32], quantum lithography [20], and now in quantum microscopy [33]. Transient entanglement of a large number of photon pairs has been shown to exist for a cascade scheme [21], and double Raman scheme [34].

The transient regime does not provide a continuous source of entangled photon pairs that could be as useful and practical as typical lasers in continuous wave operation. One might wonder whether the entanglement still survives in the long time limit.

The transient regime where the coupling varies rapidly with time is of interest. The generalization from the constant coupling to arbitrary time dependent coupling enables us to describe several new physical situations not discussed before. A realization of particular interest when may be the time-dependent alignment or orientation of the atomic/molecular dipole moment using a laser pulse [35] and motion of the atom through the cavity. More recently, we have investigated the geometric phase and entanglement of three-level double system under Raman process with and without photonic band gab (PBG) [36]. It is found that the geometric phase and entanglement are very sensitive to PBG effect, exhibiting sufficient quantum phenomena. In the present paper, we consider a single four-level atom illuminated by the pump and drive lasers as shown in Fig. 1. This scheme leads to generate controllable non-classical correlated Stokes and anti-Stokes photons from an atom [33] as well as extended medium [37]. Here, we develop the model of the four-level double Raman pairs by exploiting the required best conditions for this system, that are feasible with real experimental realization. We will explore the relationship between the quantum quantifiers for the pair of Stokes and anti-Stokes photons in terms of the physical parameters, including nonlocal correlation, physical properties, and geometric phase with and without time-dependent coupling effect.

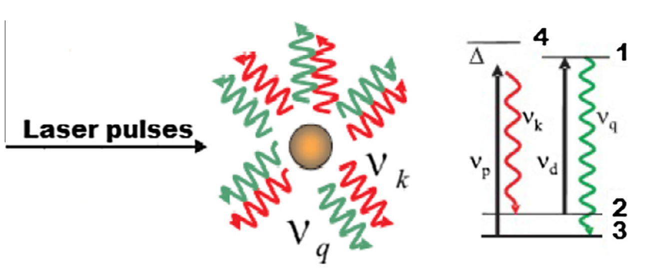

Fig. 1. An atom with four-level double Raman pairs, when illuminated by a pump and drive laser, emits a pair of Stokes and anti-Stokes photons. The atom is initially taken in the ground state (level 3). The pump field Ω_p which is detuned from $3 \leftrightarrow 4$ transition Δ, produces the Stokes photon and the control field Ω that is on resonance with $1 \leftrightarrow 2$ transition drives the atom to level 1 producing the anti-Stokes photon. The detunings are defined as $\nu_d = \omega_{12}$, $\nu_p = \Delta + \omega_{43}$, and $\nu_{k(q)} = \Delta_{k(q)} + \omega_{42(23)}$.

Previous works on single atom driven by continuous wave lasers were based on Schrodinger's equation [38–40] and master equation [41]. The former approach yields solutions only for c.w. lasers. The latter uses the correlation between the atom and field to obtain quantum regression theorem [42]. A general and more transparent method for computing correlations is based on Heisenberg–Langevin (HL) formulation, which can be extended to include spatial propagation. However, previous studies have not considered excitations with laser pulses which provide extra degrees of freedom for coherent control of non-classicality. It is interesting to consider the effects of pulse width (duration) and other parameters such as chirp, phase, and shape on quantum features of the photon pairs, as well as on the role of quantum noise [43]. Nonclassical photon pairs driven by laser pulses could be made sufficiently intense and serve as a new tool for doing quantum nonlinear optics. A scheme to generate single-cycle photon pairs has been proposed [44]. For laser pulses, exact solutions cannot be obtained using the Schrodinger's equation. The quantum regression approach may give numerical solutions for pulses, but cannot account for spatial propagation.

The paper is organized as follows. In Section "The model of two Raman photons with time-dependent coupling", we present our coupling scheme of an atom with four-level double Raman scheme driven by pump and control laser pulses, producing Stokes and anti-Stokes photons. In Section "Quantum quantifiers", we define the different quantum quantifiers, especially, von Neumann entropy as a measure of entanglement, Mandel's parameter to study the non-classical properties, and geometric phase quantity for the two-photon state. In Section "Numerical results and discussion", we analyze the results with and without time-dependent coupling effect considering two kinds of the time-dependent coupling. Finally, the main conclusions are summarized in Section "Conclusion".

The model of two Raman photons with time-dependent coupling

We consider a single four-level atom in the framework of the double Raman photons scheme, that are generally detuned to the Stokes ($\hat{a}_k$) and anti-Stokes ($\hat{a}_q$) modes, as shown in Fig. 1. The usual Hamiltonian for the atom interacting with radiation fields of two lasers is given by [36]

$$\begin{aligned}\hat{H}_{\text{int}}(t) = {} & \hbar \sum_{s=2,3} \sum_{k} \{ g_k^{s*}(t) \hat{a}_k |4\rangle\langle s| \exp\left[i(k \cdot r - \Delta_k t)\right] + H.C. \} \\ & + \hbar \sum_{r=2,3} \sum_{q} \left\{ g_q^{r*}(t) \hat{a}_q |1\rangle\langle r| \exp\left[i(q \cdot r - \Delta_q t)\right] + H.C. \right\} \\ & + \hbar \left\{ \Omega_p^* |4\rangle\langle 3| \exp\left[i(k_p \cdot r - \Delta t)\right] + \Omega^* |1\rangle \right. \\ & \left. \langle 2| \exp\left[i(k_3 \cdot r - \Delta_3 t)\right] + H.c. \right\},\end{aligned} \tag{1}$$

where $\Delta_k = \nu_k - \omega_{4s}$, $\Delta_q = \nu_q - \omega_{1s}$, $s, r = 2, 3$, $\Delta = \nu_p - \omega_{13}$, and $\Delta_d = \nu_d - \omega_{12}$ are the detunings of the Stokes and anti-Stokes frequencies. The terms $g_k^*(t) = g_k^* f(t)$ and $g_q^*(t) = g_q^* g(t)$ are the usual time-dependent coupling between the atom with Stokes and anti-Stokes modes, respectively. Since $|\Omega_p| >> |g_k^3|$ and $|\Omega| >> |g_q^b|$ for typical lasers, the first two lines of the Hamiltonian in Eq. (1) can be simplified to

$$\begin{aligned}\hat{H}_{\text{int}}(t) = \hbar & \left\{ \sum_k \left(g_k^*(t) \hat{a}_k |4\rangle\langle 2| \exp\left[i(k \cdot r - \Delta_k t)\right] \right) \right. \\ & \left. + \sum_q \left(g_q^*(t) \hat{a}_q |1\rangle\langle 3| \exp\left[i(q \cdot r - \Delta_q t)\right] \right) \right\} + H.c.\end{aligned} \tag{2}$$