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Calculation of the correlation coefficients between the numbers of counts (peak areas and backgrounds) obtained from gamma-ray spectra

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HIGHLIGHTS

- The correlation coefficients between areas of closely spaced peaks are assessed.
- For isolated peaks the correlation arises from the common continuous background.
- If peaks overlap the correlation coefficient depends on how much they overlap.
- If peaks overlap also the background height affects the correlation coefficient.
- The correlation coefficient between the peak area and its background is -1.

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ABSTRACT

Two simple methods for calculating the correlations between peaks appearing in gamma-ray spectra are described. We show how the areas are correlated when the peaks do not overlap, but the spectral regions used for the calculation of the background below the peaks do. When the peaks overlap, the correlation can be stronger than in the case of the non-overlapping peaks. The methods presented are simplified to the extent of allowing their implementation with manual calculations. They are intended for practitioners as additional tools to be used when the correlations between the areas of the peaks in the gamma-ray spectra are to be calculated. Also, the correlation coefficient between the number of counts in the peak and the number of counts in the continuous background below the peak is derived.

1. Introduction

In gamma-ray spectrometry the peak areas represent the indications that are the inputs for the measurement function used for the calculation of the measurand (ISO, 2010). The peak areas are calculated from the contents of the channels within the region of interest, comprising the corresponding peaks and their immediate vicinity. It is clear that the peak areas that are calculated from non-overlapping regions of interest cannot be correlated. However, since the region of interest comprises, besides the peak region, the spectral regions used for determining the continuous background below the peak (Gilmore, 2008), the regions of interest for the neighbouring peaks may overlap although the corresponding peak regions do not. It is easy to see that in this case the peak areas are correlated, although the peaks do not overlap. Namely, the correlation between the peak areas originates in the spectral region common to both regions of interest, which is used in the evaluation of both peak areas.

If the peaks overlap, the correlation among the peak areas becomes stronger. Since the total number of counts in the region of the composite peak is fixed, any count attributed to one peak cannot contribute to the other; therefore, in this situation a negative correlation occurs. The degree of the correlation is given by the degree of overlapping: the areas of the peaks lying close together are correlated more strongly than the areas of the peaks that are further apart. The areas of the peaks that are too close to be resolved are correlated even more strongly, but since in such a case the individual areas are not known, the value of the correlation coefficient also cannot be known. However, since any peak overlaps completely with the continuous background where it resides, a perfect negative correlation exists between the number

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of counts in the peak and the number of counts in the continuous background within its peak region (Korun et al., 2016).

In peak-analysis reports, besides the peak areas, their uncertainties are given, but the correlation coefficients are usually not (Canberra, 2013). It is therefore convenient to know the methods for how to calculate the value of the correlation coefficients between the peak areas and the correlation coefficient between the area of the peak and the number of counts in the continuous background below it. In this paper we present methods for calculating the correlation coefficients between the areas of neighbouring peaks and the derivation of the value of the correlation coefficient between the number of counts in a peak and the number of counts in the continuous background below it.

2. Methods

2.1. Correlation coefficients between areas of non-overlapping peaks

The area of an isolated peak is calculated as (Gilmore, 2008)

$$N_n = N_T - \frac{n}{n_< + n_>}(N_< + N_>), \tag{1}$$

where N_n denotes the net number of counts in the peak (the peak area), N_T is the total number of counts within the peak region, $N_<$ is the number of counts in the spectral region adjacent to the peak region on the low-energy side and $N_>$ is the number of counts in the spectral region adjacent to the high-energy side of the peak. n, $n_<$ and $n_>$ are the numbers of channels in the peak region and in the adjacent regions at the low- and high-energy sides of the peak region, used for interpolating the continuous background below the peak respectively.

When the regions of interest for neighbouring peaks overlap, but the peak regions do not, then the adjacent region on the high-energy side of the peak at the lower energy overlaps with the adjacent region on the low-energy side of the peak at the higher energy. The peak areas of the neighbouring peaks are

$$N_{n1} = N_{T1} - \frac{n_1}{n_{<1} + n_{>1}}(N_{<1} + N_{>1}' + N_O) \tag{2}$$

and

$$N_{n2} = N_{T2} - \frac{n_2}{n_{<2} + n_{>2}}(N_{<2}' + N_O + N_{>2}), \tag{3}$$

where the indices 1 and 2 refer to the peak at the lower and the peak at the higher energy, respectively. Here, $N_{>1}'$ and $N_{<2}'$ denote the number of counts in the regions $n_{>1}$ and $n_{<2}$ that do not overlap and N_O is the number of counts in the region of the overlap. Because the adjacent regions overlap it follows

$$n_O = n_{>1} - n_{>1}' = n_{<2} - n_{<2}', \tag{4}$$

where n_O denotes the number of channels that belong to both adjacent regions (Fig. 1) and $n_{>1}'$ and $n_{<2}'$ are the numbers of channels in both regions that are outside the overlapping range. The numbers of counts in these regions are related by

$$N_O = N_{>1} - N_{>1}' = N_{<2} - N_{<2}'. \tag{5}$$

Since N_O appears in the expressions for N_{n1} and N_{n2} in Eqs. (2) and (3), respectively, it is evident that the quantity N_O introduces the correlation between both peak areas. The uncertainties of these peak areas are

$$u^2(N_{n1}) = N_{T1} + \left(\frac{n_1}{n_{<1} + n_{>1}}\right)^2(N_{<1} + N_{>1}' + N_O) \tag{6}$$

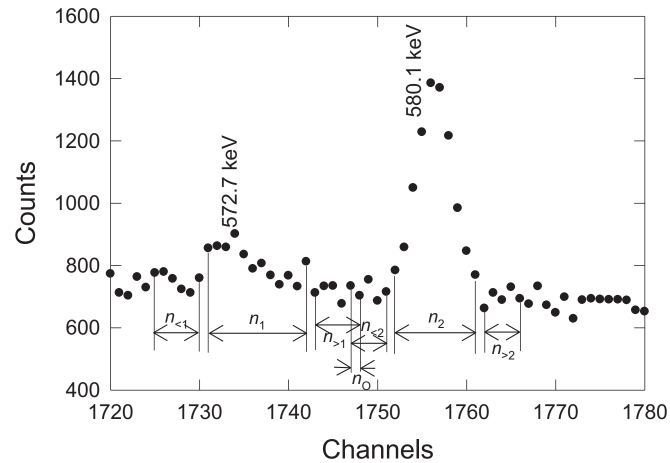

Fig. 1. The spectral region of two isolated peaks having weakly overlapping regions used for determining the height of the continuous background.

and

$$u^2(N_{n2}) = N_{T2} + \left(\frac{n_2}{n_{<2} + n_{>2}}\right)^2(N_{<2}' + N_O + N_{>2}). \tag{7}$$

It follows from Eqs. (6) and (7) that the peak-area uncertainties can be separated into the correlated parts comprising the common source of the uncertainty

$$u_C^2(N_{n1}) = \left(\frac{n_1}{n_{<1} + n_{>1}}\right)^2 N_O \tag{8}$$

and

$$u_C^2(N_{n2}) = \left(\frac{n_2}{n_{<2} + n_{>2}}\right)^2 N_O \tag{9}$$

and into the uncorrelated parts

$$u_N^2(N_{n1}) = N_{T1} + \left(\frac{n_1}{n_{<1} + n_{>1}}\right)^2(N_{<1} + N_{>1}') \tag{10}$$

and

$$u_N^2(N_{n2}) = N_{T2} + \left(\frac{n_2}{n_{<2} + n_{>2}}\right)^2(N_{<2}' + N_{>2}). \tag{11}$$

The correlation coefficient between N_{n1} and N_{n2} is given as (Glavič-Cindro et al., 2004):

$$r(N_{n1}, N_{n2}) = \frac{u_C(N_{n1})u_C(N_{n2})}{u(N_{n1})u(N_{n2})}. \tag{12}$$

It is easy to see that the correlation coefficient is non-negative and that it is an increasing function of N_O, the number of counts that are common to both regions of interest. It attains a value of 0.25 in the case of a dominating and constant background, when $n_O = n_{>1} = n_{<2}$ and when $n_{<1} = n_{>1}$, $n_{<2} = n_{>2}$ and $n_1 = n_2 = n_{<1} + n_{>1}$.

2.2. Correlation coefficients between the areas of overlapping peaks

Usually, the method of least squares is used when overlapping peaks are decomposed in the individual peaks (Debertin and Helmer, 1988). This calculation cannot be performed manually (Gilmore, 2008); therefore, neither the peak areas nor their uncertainties can be arrived at through manual calculations. To render a manual calculation of the correlation coefficient possible,