Applied Radiation and Isotopes 107 (2016) 152–159

Contents lists available at ScienceDirect

Applied Radiation and Isotopes

journal homepage: www.elsevier.com/locate/apradiso

Semi-empirical model for fluorescence lines evaluation in diagnostic x-ray beams

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HIGHLIGHTS

- A semi-empirical model of fluorescence lines evaluation is proposed.
- Indirect radiation interaction is discussed.
- A semi-model of direct radiation interaction is presented and described.
- Experimental tests are presented and used to calibrate the model parameters.
- A comparison with independent data taken from the literature is discussed.

ARTICLE INFO

Article history:
Received 9 May 2014
Received in revised form
14 October 2015
Accepted 15 October 2015

PACS:
87.64.Aa
29.30.Kv
32.80.Aa
89.20.Bb

Keywords:
x-ray
Bremsstrahlung
Fluorescence lines
Direct radiation
Indirect radiation

ABSTRACT

Diagnostic x-ray beams are composed of bremsstrahlung and discrete fluorescence lines. The aim of this study is the development of an efficient model for the evaluation of the fluorescence lines. The most important electron ionization models are analyzed and implemented. The model results were compared with experimental data and with other independent spectra presented in the literature. The implemented peak models allow the discrimination between direct and indirect radiation emitted from tungsten anodes. The comparison with the independent literature spectra indicated a good agreement.

1. Introduction

Since its discovery, the x-radiation became a very useful tool. Several imaging applications, such as conventional and interventionist radiology, computed tomography and mammography, use x-ray polychromatic beams composed of a bremsstrahlung continuous component together with fluorescence lines.

Many attempts for computing theoretical x-ray spectrum were developed and enhanced over time. However, few rigorous calculation method was developed to evaluate the characteristic radiation, possibly due to its relatively small contribution (10–15%) in comparison with the bremsstrahlung component (Birch and Marshall, 1979; Tucker et al., 1991; Poludniowski, 2007). Indeed, the three mathematical models more consistently validated for diagnostic x-ray spectra calculations are focused on the

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http://dx.doi.org/10.1016/j.apradiso.2015.10.018

bremsstrahlung and only provide a rough description of characteristic radiation in terms of completeness (Birch and Marshall, 1979; Tucker et al., 1991; Poludniowski and Evans, 2007; Poludniowski, 2007).

Poludniowski (2007) described the x-ray spectrum and developed a calculation of the characteristic radiation that was based on a semi-empirical description of the *indirect* radiation component for tungsten anodes.

The model developed by Birch and Marshall (1979) is considered accurate and the faster model for simulating x-ray spectrum via Monte Carlo algorithms (Bontempi et al., 2010). It uses the Green and Cosslett (1968) empirical model to describe tungsten characteristic radiation as a function of the tube voltage. Birch and Marshall also used the Storm and Israel (1970) Data Tables to calculate the energies and the relative intensities of K and L fluorescence lines. The other model, developed by Tucker et al. (1991), modified by Costa et al. (2007), uses the same empirical description adjusting the exponent value (1.67 as opposed to 1.63) according to Green and Cosslett (1961) theoretical predictions. It then incorporates the terms studied by Vignes and Dez (1968) to take into account the depth of production of the characteristic x-rays. Pella et al. (1985) implemented an algorithm to evaluate the fluorescence lines of various materials starting from the work of Green and Cosslett (1961) and calculating the ratio between characteristic radiation and bremsstrahlung continuum component.

Although the peak models discussed treat both the direct and indirect radiations, their formulations are based on a very old x-ray model (Kramers, 1923) that has many approximations and strong hypotheses that make it unusable for calculating real x-ray spectra.

Moreover, the direct characteristic radiation description proposed by Green and Cosslett (1961) is derived from the non-relativistic ionization cross section of Bethe (1930). So the presented models are based on approximations. They are valid but old, and sometimes obsolete.

Thus the aim of this work is to analyze the contribution of fluorescent radiation using modern direct and indirect radiation models. Three major models of electron ionization cross section will be analyzed and implemented and the most effective and accurate will be chosen and used to simulate x-ray characteristic radiation.

2. Materials and methods

The description of direct and indirect characteristic radiation starts from the observation of the physical phenomena that happen inside the x-ray tube. There are three major events that can happen: electron ionization (*direct radiation*), Auger electron emission (negligible, Dyson, 1990) and photoelectric ionization and vacancy fill (*indirect radiation*). Together with the bremsstrahlung photon production, these phenomena contribute to make the x-ray spectrum:

$$n_{sp}(E) = Qf(E)\left(n_{br}(E) + \sum_{j} \delta(E - E_j) n_j\right) \tag{1}$$

where Q takes into account the source–detector distance, the beam collimation, the tube current, the exposure time and the detection efficiency. The term $f(E)$ indicates the total filtration of the x-ray tube (housing/inherent plus additional filtration) and depends on the photon energies. The functions n_{br} and n_j refer to the counts of bremsstrahlung and characteristic photons. The Dirac delta function, $\delta(E - E_j)$, selects the peak position in terms of the energy to be added to the bremsstrahlung continuum. In an experimental spectrum, the δ appears as a bell-shaped peak that depends on the energy resolution of the instruments used for detecting the spectra.

In order to clarify the notations used, the symbol "E" refers to photon energy, "T" is used to indicate the electron kinetic energy and "kVp" indicates the value of the x-ray tube voltage. The subscript "k" then refers to the k-th atomic shell (K, L, M, N) with energy E_k while the subscript "j" refers to the j-th characteristic peak (es: $K_{\alpha1}$, or $K_{\alpha2}$, etc.) with centroid energy E_j.

2.1. Indirect radiation

One of the most robust models for accounting the production of the *indirect* characteristic radiation was proposed by Poludniowski (2007).

The amount of characteristic photons was evaluated considering that the ratio of direct and indirect radiation is constant over a wide range of tube voltages, as stated by Dyson (1990). According to the Poludniowski model, the intensity of the indirect radiation is

$$n_j^{ind} = p_j f_j A_k N_{br}^{emit} \tag{2}$$

where p_j is the intensity of the j-th peak (Thompson et al., 2009), f_j is the total filtration of the tube at the j-th peak energy, and A_k is the probability that a bremsstrahlung photon will appear as indirect radiation (Poludniowski, 2007). $N_{br}{}^{emit}$ is the total number of bremsstrahlung photons emitted with energies $E \geq E_k$:

$$N_{br}^{emit} = Q \int_{E_k}^{kVp} n_{br}(E)\, dE \tag{3}$$

Then Poludniowski described the contribution of the term A_k, reaching the final formulation of the total characteristic radiation:

$$n_j = (1 + p_d) n_j^{ind} \tag{4}$$

where p_d is the ratio between direct and indirect radiation.

2.2. Direct radiation

As previously stated, the other component of the characteristic radiation is the direct interaction of the cathode-emitted electrons with the orbital electrons into the anode material. This interaction could ionize the anode atoms and generate photons with a well-defined energy.

The model of Green and Cosslett (1961) was based on the equation

$$n_k^{dir} = \frac{\rho N_A}{A} Q \int_{E_k}^{kVp} \sigma_k(T) \left(\frac{dT}{dx}\right)^{-1} dT \tag{5}$$

where $\frac{\rho N_A}{A}$ is the number of atoms per volume unit in the target, ρ is the density of the target, N_A is Avogadro's number, A is the atomic weight of the target and σ_k is the ionization cross section at the k-th shell and $\frac{dT}{dx}$ is the electron stopping power inside the anode. This equation does not take into account the auto-absorption of the generated photons inside the anode and the parameter σ_k was based on the Bethe (1930) non-relativistic electron-ionization cross section.

On the basis of the models of Birch and Marshall (1979) and Tucker et al. (1991), this equation can be updated adding the auto-absorption of the anode. Thus the final form of the direct characteristic radiation is

$$n_j^{dir} = Qf_j p_j B_k \left[\frac{\rho N_A}{A} \int_{E_k}^{kVp} \sigma_k(T) e^{-\mu_j d_T} \left(\frac{dT}{dx}\right)^{-1} dT \right] \tag{6}$$