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Electron impact ionization of metastable 2P-state hydrogen atoms in the coplanar geometry

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ABSTRACT

Triple differential cross sections (TDCS) for the ionization of metastable 2P-state hydrogen atoms by electrons are calculated for various kinematic conditions in the asymmetric coplanar geometry. In this calculation, the final state is described by a multiple-scattering theory for ionization of hydrogen atoms by electrons. Results show qualitative agreement with the available experimental data and those of other theoretical computational results for ionization of hydrogen atoms from ground state, and our first Born results. There is no available other theoretical results and experimental data for ionization of hydrogen atoms from the 2P state. The present study offers a wide scope for the experimental study for ionization of hydrogen atoms from the metastable 2P state.

1. Introduction

The study of multiple ionization process by charged particle impact is of great interest in many branches of physics, such as astrophysics, plasma physics and also in life sciences, for understanding the various mechanisms leading to energy deposition by radiation of matter.

Ionization of the hydrogen atoms by electrons is a good check for the perturbation theory, because of the existence of experimental results, specially for the Triple differential cross sections (TDCS) [1–5]. Ionization by fast particles was first treated quantum mechanically by Bethe [6]. Electron impact of single and double ionization is one of the simplest and the most important fundamentals of such a process. The utilization of the multi-parameter detection technique, together with the progress in computational methods, have made it possible to perform a complete experiment in which kinematical parameters (like momentum and energies) of all acting particles are determined. In such calculations, the ejected electron is detected in coincidence with the scattered electrons and it is a well known experiment [4]. This kind of experiment, called (e, 2e) experiments, have been successfully used during the last four decades to investigate the fine details of the ionization process both in the ground state [7–16] and metastable [17–28] states of atomic Hydrogen.

Hafid et al. [20] have shown that the application to the H(2S) of the corrected double continuum wave function of Brauner et al. [29] gives results comparable to the second Born ones. In the BBK theory of Brauner et al. [29] focused attention was seen on the improvement of the final state wave function by including in it the effects of all the long range Coulomb interactions, including the electron–electron repulsion. This satisfies the correct boundary condition when the particle separations tend to infinity.

For the ionization of hydrogen atoms by electrons from the metastable 2S and 2P states, no such triple differential cross sections (TDCS) measurement is yet available in the literature, although the absolute total cross sections (TCS) were measured much earlier [30,31]. Recently, Dal et al. [25] has investigated in a greater detail the ionization of atomic hydrogen and helium atoms in the different metastable states by electrons and positrons. In this study we have investigated the ionization of metastable 2P state hydrogen atoms by electrons. To the best of our knowledge, the work reported here, is introducing the TDCS calculation for the ionization of metastable 2P-state hydrogen atoms by electrons for the first time.

A multiple scattering theory [10,11] has been followed in the present calculation of the triple differential cross sections (TDCS) in the metastable 2P-state hydrogen atom ionization by 250 eV electron energy. It is noted that the multiple scattering wave function [11] has been designed for two electrons moving in a coulomb field, which include higher order and correlation effects. Using this wave function, very interesting results for triple differential cross sections (TDCS) with various kinematic conditions have been obtained for electron hydrogen ionization collisions both in the ground state and metastable 2S state at non-relativistic energies [11,13,21,22] and many other calculations (the references of which

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are not given here) as well as for medium-heavy atoms ionization by electrons at relativistic energies [23,24,32,33]. So it will be interesting here to use this wave function in the present study for ionization of metastable 2P state hydrogen atoms by electrons.

2. Theory

2.1. Scattering mechanism

The most detailed information presently available is about the single ionization processes of the following type

$$e^- + H(2P) \rightarrow H^+ + 2e^- \tag{1}$$

where the symbol 2P denotes the metastable state of the hydrogen atoms, and has been obtained in the coplanar geometry by analyzing triple differential cross sections (TDCS) measured in (e, 2e) coincidence experiments. The TDCS is a measure of the probability that in an (e, 2e) reaction an incident electron of momentum $\bar{p}_i$ and energy E_i will produce on collision with the target two electrons having energies E_1 and E_2 and momenta $\bar{p}_1$ and $\bar{p}_2$, emitted respectively into the solid angles $d\Omega_1$ and $d\Omega_2$ centered about the directions (θ_1, ϕ_1) and (θ_2, ϕ_2).

The TDCS is usually denoted by the symbol $d^3\sigma/d\Omega_1 d\Omega_2 dE_2$ for unpolarized incident electrons and targets, it is a function of the quantities E_i, E_1 or E_2, θ_1, θ_2 and $\phi = \phi_1 - \phi_2$. By integrating the TDCS over $d\Omega_1$, $d\Omega_2$ or dE_2 one can form various double and single differential cross sections. We will calculate the same in the near future. Finally, the total ionization cross section is obtained by integrating over all outgoing scattering angles and energies, and depends only in E_i, the incident electron energy. It is useful when studying (e, 2e) coincidence experiments to distinguish between several kinematical arrangements, since these have important implications for the theoretical analysis of the collision. A first distinction can be made between coplanar geometries-such that the momenta $\bar{p}_i$, $\bar{p}_1$ and $\bar{p}_2$ are in the same plane-and non-coplanar geometries such that the momentum $\bar{p}_2$ is out of the $(\bar{p}_i, \bar{p}_1)$ reference plane. There is another useful distinction between asymmetric and symmetric geometries. In asymmetric geometries, a fast electron of energy E_i is incident on the target atom, and a fast ("scattered") electron is detected in coincidence with a slow ("ejected") electron. This kind of experiment was first performed by Ehrhardt et al. [4]. On the other hand, symmetric geometries are defined by the requirement that $\theta_1 \cong \theta_2$ and $E_1 \cong E_2$.The first (e, 2e) symmetric coincidence experiments of Amaldi et al. [34] have been followed by a number of experiments of this type.

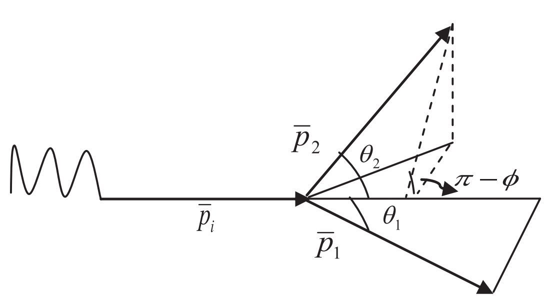

Fig. The kinematics of an (e, 2e) reaction. The incident electron momentum is $\bar{p}_i$ and the momenta of the outgoing electrons are $\bar{p}_1, \bar{p}_2$ respectively. Also the angles θ_1 and θ_2 are shown with respect to the incident direction and the angle $\pi - \phi$ is measuring the direction in a coplanar situation.

2.2. The T-matrix element

The direct T-matrix element for ionization of hydrogen atoms by electrons [11] may be written as

$$T_{fi} = \langle \Psi_f^{(-)}(\bar{r}_1, \bar{r}_2) | V_i(\bar{r}_1, \bar{r}_2) | \phi_i(\bar{r}_1, \bar{r}_2) \rangle \tag{2}$$

where the perturbation potential $V_i(\bar{r}_1, \bar{r}_2)$ is given by

$$V_i(\bar{r}_1, \bar{r}_2) = \frac{1}{r_{12}} - \frac{Z}{r_2}$$

For hydrogen atoms nuclear charge is $Z = 1$, r_1 and r_2 are the distances of the two electrons from the nucleus and r_{12} is the distance between the two electrons.

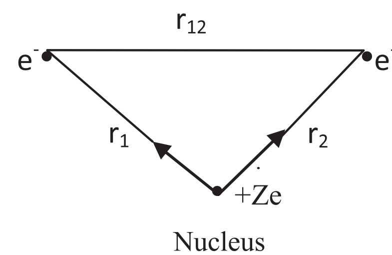

Fig. Interaction between two electrons and the nucleus.

The initial channel unperturbed wave function is given by

$$\phi_i(\bar{r}_1, \bar{r}_2) = \frac{e^{i\bar{p}_i.\bar{r}_2}}{(2\pi)^{3/2}} \phi_{2p}(\bar{r}_1) = \frac{e^{i\bar{p}_i.\bar{r}_2}}{8\sqrt{2}\pi^2} r_1 \cos\theta e^{-r_1\lambda_1} \tag{3}$$

where

$$\phi_{2p}(\bar{r}_1) = \sqrt{\frac{1}{32\pi}} r_1 \cos\theta e^{-r_1/2} = \sqrt{\frac{1}{32\pi}} r_1 \cos\theta e^{-\lambda_1 r_1} \quad \left[\lambda_1 = \frac{1}{2}\right]$$

is the hydrogenic 2P state wave function, $\bar{p}_i$ is the incident electron momentum.

And $\psi_f^{(-)}(\bar{r}_1, \bar{r}_2)$ is the final three-particle scattering state wave function with the electrons being in the continuum with momenta $\bar{p}_1$ and $\bar{p}_2$. Co-ordinates of the two electrons are taken to be $\bar{r}_1$ and $\bar{r}_2$. Here the approximate wave function $\psi_f^{(-)}(\bar{r}_1, \bar{r}_2)$ [11] is given by

$$\begin{aligned}\psi_f^{(-)}(\bar{r}_1, \bar{r}_2) = N(\bar{p}_1, \bar{p}_2)\Big[&\phi_{\bar{p}_1}^{(-)}(\bar{r}_1) e^{i\bar{p}_2.\bar{r}_2} + \phi_{\bar{p}_2}^{(-)}(\bar{r}_2) e^{i\bar{p}_1.\bar{r}_1} + \phi_{\bar{p}}^{(-)}(\bar{r}) e^{i\bar{P}.\bar{R}} \\ &-2e^{i\bar{p}_1.\bar{r}_1 + i\bar{p}_2.\bar{r}_2}\Big] \Big/ (2\pi)^3\end{aligned} \tag{4}$$

where

$$\bar{r} = \frac{\bar{r}_1 - \bar{r}_2}{2}, \quad \bar{R} = \bar{r}_1 + \bar{r}_2, \quad \bar{p} = (\bar{p}_2 - \bar{p}_1), \quad \bar{P} = \bar{p}_2 + \bar{p}_1$$

The scattering amplitude [11] may be written as

$$f(\bar{p}_1, \bar{p}_2) = N(\bar{p}_1, \bar{p}_2)[f_{eT} + f_{PT} + f_{Pe} - 2f_{PWB}] \tag{5}$$

where f_{eT}, f_{PT}, f_{Pe} and f_{PWB} are the amplitudes corresponding to the four terms of Eq. (4) respectively.

The normalization constant $N(\bar{p}_1, \bar{p}_2)$ is given by

$$\begin{aligned}|N(\bar{p}_1, \bar{p}_2)|^{-2} = \Bigg| 7 - 2[\lambda_1 + \lambda_2 + \lambda_3] - \left[\frac{2}{\lambda_1} + \frac{2}{\lambda_2} + \frac{2}{\lambda_3}\right] \\ + \left[\frac{\lambda_1}{\lambda_2} + \frac{\lambda_1}{\lambda_3} + \frac{\lambda_2}{\lambda_1} + \frac{\lambda_2}{\lambda_3} + \frac{\lambda_3}{\lambda_1} + \frac{\lambda_3}{\lambda_2}\right]\Bigg|\end{aligned} \tag{6}$$

where

$$\begin{aligned}\lambda_1 &= e^{\pi\alpha_1/2}\Gamma(1 - i\alpha_1), \quad \alpha_1 = 1/p_1, \\ \lambda_2 &= e^{\pi\alpha_2/2}\Gamma(1 - i\alpha_2), \quad \alpha_2 = 1/p_2, \\ \lambda_3 &= e^{\pi\alpha/2}\Gamma(1 - i\alpha) \qquad \alpha = -1/p.\end{aligned}$$