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Inertial-Hall effect: the influence of rotation on the Hall conductivity

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ABSTRACT

Inertial effects play an important role in classical mechanics but have been largely overlooked in quantum mechanics. Nevertheless, the analogy between inertial forces on mass particles and electromagnetic forces on charged particles is not new. In this paper, we consider a rotating non-interacting planar two-dimensional electron gas with a perpendicular uniform magnetic field and investigate the effects of the rotation in the Hall conductivity. The rotation introduces a shift and a split in the Landau levels. As a consequence of the break of the degeneracy, the counting of the states fully occupied below the Fermi energy increases, tuning the Hall quantization steps. The rotation also changes the quantum Hall plateau widths. Additionally, we find the Hall quantization steps as a function of rotation at a fixed value of the magnetic field.

Introduction

Since the discovery of the integer quantum Hall effect (IQHE) in 1980 by von Klitzing et al. [1], the two-dimensional electron gas (2DEG) in a strong perpendicular magnetic field has been a subject of intense study, both experimentally and theoretically. The IQHE is a macroscopic effect of solid state physics and it is characterized by a quantized Hall conductivity which is given by integer multiples of e^2/h, where e is the electrical charge and h is Planck's constant. The quantization of the Hall conductivity has been measured to 1 part in 10^9 [2,3]. This precision reveals the topological nature of the Hall conductivity, which does not depend on the material, geometry and microscopic details of the sample, and makes the IQHE very useful in the field of metrology. The properties of a charged particle in a magnetic field are important also in other fields as high energy physics, atomic physics and astrophysics, as was pointed out in [4], which has attracted even more interest in the study of the IQHE.

The Coriolis force acts on a particle of mass m very much like the magnetic force on a charged particle. This analogy has been explored by Aharonov and Carmi [5,6] in the early 1970s, by Sakurai [7] in 1980 and by Tsai and Neilson [8] in 1988 in the context of a rotational quantum phase similar to Aharonov–Bohm's. The idea of rotation working as an effective magnetic field is in fact quite old. In 1915 Barnett [9,10] already published a paper on magnetization by rotation which has recently had a renewed interest applied to nanostructures [11,12]. A rotational analog of the classical Hall effect has been proposed [13] and the inertial effects of rotation in spintronics studied [14–16]. Based on the same analogy, Dattoli and Quattromini [17], introduced Coriolis quantum states analogous to Landau levels (LLs). This analogy also appears in the study of rapidly rotating Bose–Einstein Condensates [18], for the Hamiltonian describing a rotating gas in a harmonic trap is similar to that for charged particles in a magnetic field. The subject of analog Landau levels has been recently approached in the more general context of combined non-inertial, gravitational and electromagnetic effects by Konno and Takahashi [19] who were interested on quantum states on the surface of a rotating star. The Quantum Hall effect under rotation has been discussed in more general grounds in [20,21].

The Coriolis force does not come alone. Its companion, the centrifugal force, will be also felt by the particle in the rotating system. Together, the Coriolis and centrifugal contributions to the quantum Hamiltonian lead only to a coupling between the particle angular momentum and the rotation, for the case of a spinless particle. This gives rise to non-degenerate, sample-length dependent, Landau levels [20,21]. Neglecting the centrifugal part, besides this coupling, there appears the richer structure of rotational Landau levels [17]. On the other hand, if we are free from the centrifugal force we end up with a Landau levels system that includes the coupling between the particle angular momentum and the rotation. It

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should be noticed that, if a steady time variation of the rotation is assumed, then the Euler force should be included in the analysis.

The system of our interest consists in a non-interacting free electron gas in a rotating planar conductor with a uniform magnetic field applied perpendicular to the rotating plane. Our purpose is to investigate the quantum Hall effect in this system, analyzing the influence of the rotation in the Hall conductivity. Charged particles in a rotating Hall sample were already studied in Ref. [21], where it was pointed out that the quantization of the Hall conductivity is not affected by the rotation. However, as it will be shown here, the rotation breaks same degeneracy of the LLs and the counting of states fully occupied below the Fermi energy may change, altering the Hall quantization steps.

The paper is organized as follows. In Section "The spinless charged particle" we write out the Hamiltonian of a charged particle in a rotating disk in the presence of a magnetic field and find the energy spectrum. In Section "Electronic structure" we analyze the electronic structure, showing that the rotation induces a shift and a split in the Landau levels. In Section "Hall conductivity" we investigate the influence of the rotation in the Hall conductivity. The paper is summarized and concluded in Section "Conclusion".

The spinless charged particle

Let us consider a free particle in a rotating disk with a uniform magnetic field perpendicular to the disk [see Fig. 1]. The Coriolis and centrifugal forces are given by

$$\vec{F}_{\rm Cor} = 2m(\vec{v}\times\vec{\Omega}), \tag{1}$$

and

$$\vec{F}_{\rm Cen} = -m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}), \tag{2}$$

respectively. These forces enter the Schrödinger Hamiltonian as a vector and scalar inertial potential [5,6] given by

$$\vec{A}_{\rm ine} = \frac{1}{2}(\vec{\Omega}\times\vec{r}) \tag{3}$$

and

$$V_{\rm ine} = -\frac{1}{2}(\vec{\Omega}\times\vec{r})^2, \tag{4}$$

respectively, and the Hamiltonian is written as

$$H = \frac{1}{2m}(\vec{p}-2m\vec{A}_{\rm ine})^2 + mV_{\rm ine}. \tag{5}$$

A magnetic field $\vec{B}$ applied in the laboratory will be felt by charged particles in the rotating reference frame as an electric and a magnetic field given by [15]

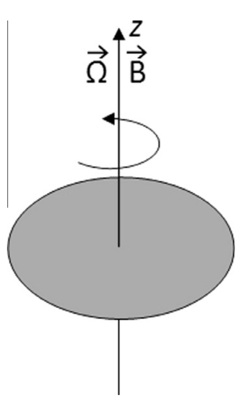

Fig. 1. A rotating disk with a perpendicular uniform magnetic field. The rotation speed and magnetic field vector are in the z direction.

$$\vec{E}' = (\vec{\Omega}\times\vec{r})\times\vec{B} \tag{6}$$

and

$$\vec{B}' = \vec{B}. \tag{7}$$

Therefore, the Hamiltonian in cylindrical coordinates of a particle in a rotating disk in the presence of a magnetic field, with $\vec{\Omega} = \Omega\hat{z}$ and $\vec{B} = B\hat{z}$, can be written as

$$H = \frac{[\vec{p}-q\vec{A}-m(\vec{\Omega}\times\vec{r})]^2}{2m} - \frac{m(\vec{\Omega}\times\vec{r})^2}{2} + qV, \tag{8}$$

where V and $\vec{A}$ are the scalar and vector electromagnetic potentials, and are given by

$$V = -\frac{\Omega B r^2}{2}, \tag{9}$$

$$\vec{A} = \left(0, \frac{Br}{2}, 0\right). \tag{10}$$

Thus, the Hamiltonian can be summarized to

$$H = \frac{p^2}{2m} - \alpha r p_\phi + \beta r^2, \tag{11}$$

with

$$\alpha = \frac{qB}{2m} + \Omega, \tag{12}$$

$$\beta = \frac{q^2B^2}{8m}. \tag{13}$$

For this Hamiltonian, the Schrödinger equation can be written as

$$-\frac{\hbar^2}{2m}\nabla^2\psi + i\alpha\hbar\frac{\partial\psi}{\partial\phi} + \beta r^2\psi = E\psi. \tag{14}$$

With the *ansatz* $\psi = R(r)e^{-il\phi}$, Eq. (14) becomes

$$r^2R'' + rR' + (-\sigma^2 r^4 + \lambda r^2 - l^2)R = 0, \tag{15}$$

where $\sigma^2 = \frac{q^2B^2}{4\hbar^2}$ and $\lambda = \frac{2m}{\hbar}\left(\frac{E}{\hbar} - \frac{qBl}{2m} - \Omega l\right)$. Writing $\sigma r^2 = \xi$ and looking to the asymptotic limits when $\xi\to\infty$ and $\xi\to 0$, one can propose a solution of the form

$$R(\xi) = e^{-\frac{\xi}{2}}\xi^{\frac{|l|}{2}}u(\xi). \tag{16}$$

Replacing it in Eq. (15), one gets

$$\xi\frac{d^2u}{d\xi^2} + [1+|l|-\xi]\frac{du}{d\xi} + \left[\frac{\lambda}{4\sigma} - \frac{1}{2}(|l|+1)\right]u = 0, \tag{17}$$

that is a confluent hypergeometric equation, which has solution

$$u = A\cdot F\left(\frac{-\lambda}{4\sigma} + \frac{1}{2}(|l|+1), 1+|l|, \xi\right), \tag{18}$$

where A is a constant and $F(a,b,z)$ is a confluent hypergeometric function, in this case, degenerate. In order to have a finite polynomial function (the hypergeometric series has to be convergent in order to have a physical solution), the condition $a = -n$ has to be satisfied, where n is a positive integer number. From this condition, the discrete possible values for the energy are given by

$$E_{n,\ell} = \hbar\omega_c\left\{n + \frac{\ell}{2} + \frac{|\ell|}{2} + \frac{1}{2}\right\} + \Omega l\hbar, \tag{19}$$

where $\omega_c = qB/m$ is the cyclotron frequency. The wave function is

$$\psi = Ae^{-il\phi-\frac{\sigma r^2}{2}}(\sigma r^2)^{\frac{|l|}{2}}\times F\left(-\frac{\lambda}{4\sigma} + \frac{1}{2}(|l|+1), 1+|l|, \sigma r^2\right). \tag{20}$$

One can notice that the values of $\vec{B}$ and $\vec{\Omega}$ are arbitrary. Therefore, one can adjust the rotation and the magnetic field for different values.