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Optimal solution of nonlinear heat and mass transfer in a two-layer flow with nano-Eyring–Powell fluid

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ABSTRACT

This paper deals with the fully-developed two-layer Eyring–Powell fluid in a vertical channel divided into two equal regions. One region is filled with the clear Eyring–Powell fluid and other with the nano-Eyring–Powell fluid. The flow is observed under the uniform wall temperature and concentration boundary conditions for combined heat and mass transfer. The governing coupled nonlinear ordinary differential equations (ODEs) of the flow in each layer are analytically solved by using optimal homotopy analysis method (OHAM) based on the homotopy analysis method (HAM). HAM is an efficient analytical approximation method to solve highly nonlinear problems. The effect of Brownian motion parameter on Eyring–Powell fluid is also observed and the influence of significant parameters is presented for their different values.

1. Introduction

The study of mixed, free and forced convective heat transfer in a vertical parallel plate channel has always gained massive attention because of its wide range applications in many industrial processes. Methodologies to suppress or eliminate interfacial instabilities and further stabilize multi-layer flows, are therefore inherently of interest. Some examples comprise microelectronic cooling, design of cooling system in electronic devices, nuclear reactors cooled during emergency shutdown, chemical processing equipment, solar technology, etc. Initially, Tao [1] has investigated the laminar fully developed mixed convection in a vertical channel and later, this work of Tao was extended by Habchi and Acharya [2] to asymmetric heating where one plate is heated and the other plate is adiabatic. Further, in a vertical channel with asymmetric wall temperature, Aung and Worku [3] analyzed developing flow and flow reversal and then they provided results for mixed convection flow with different wall temperatures [4]. Single-fluid model was considered in all the above mentioned studies, but a large amount of the scientific and technological problems related to plasma physics, petroleum industry, magnetofluid dynamics, geophysics, etc., involve multifluid flow situations. Multilayer flows mainly occur in three different patterns. First of all in co-extrusion processes which make a product of more than one layer concurrently. Secondly, several film coating processes involve a multi-layer, where on each fluid substrate a different layer is used. Thirdly, in lubricated transport processes in which between the walls of a duct and the transported fluid, a lubricating fluid lies in a layer. The flow and the heat transfer of two immiscible fluids were investigated by Nikodijevic et al. [5] in the presence of a uniform inclined magnetic field. And a three-layer unsteady flow in which porous media are sandwiched between viscous fluids was studied by Umavathi et al. [6]. Recently, Farooq et al. investigated the two-layer flow with nanofluids for third grade-fluids [7]. They have considered the mixed-convection in a vertical channel and the fluid properties at the interface are also observed.

The study of nanofluids has now become a global research area and it has gained the massive attention of researchers in the last few years. Its properties are known to be effective on heat transfer and convective flows such as viscosity and thermal conductivity. Water, oil and ethylene glycol mixture are the conventional heat transfer fluids and these fluids are poor at heat transfer. An innovative technique has been used extensively to improve the heat transfer by using ultra fine solid particles in the fluids during the last decade. For this purpose, Choi [8] introduced the term nanofluid which refers to the fluids by suspending nano-scale (less than 1%) particles in the base fluid which increases the thermal conductivity of the fluid up to approximately two times. Nanotechnology is regarded as one of the most significant forces that is the foundation of the next major industrial revolution of this century and it causes many applications in space crafts, electronic devices, artificial organs, metrology and cooling applications of nanofluids,

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etc. Therefore, nanofluids promise to fetch about a revolution in cooling technologies. As a consequence of these discoveries, research and development on nanofluids has drawn considerable attention from industry and academia over the past several years. For the first time Khanafer et al. [9] examined heat transfer performance of nanofluids enclosing the solid particle dispersion. A detailed review on nanofluids has been given in a book entitled Nanofluids: Science and Technology by Das et al. [10]. For a fully developed flow the effect of nanoparticle volume fraction on velocity and temperature distribution was studied by Xu and Pop [11]. They have also counted into the laminar mixed convection flow of a nanofluid caused by both the buoyancy force and external pressure gradient in a vertical channel [12]. Nadeem et al. [13] inspected the non-aligned stagnation point nano fluid over a convective surface in the presence of a partial slip. Furthermore, they numerically investigated the effect of the magnetic field on the oblique flow of a Walter-B type nano fluid over a convective surface [14].

Eyring–Powell fluid model [15] a complete mathematical model was proposed by Powell and Eyring in 1944. It possesses many advantages over the non-Newtonian fluid models such as it is evoked from the kinetic theory of liquids rather than the empirical relation and also for low and high shear rates it correctly reduces to Newtonian behavior. The flow of an Eyring–Powell model fluid due to a stretching cylinder with variable viscosity under boundary layer conditions was presented by Malik et al. [16]. Hayat et al. [17] examined the steady flow of an Eyring–Powell fluid over a striking surface with convective boundary conditions. Ara et al. [18] investigated the effect of thermal radiations on this flow over an exponentially shrinking sheet.

To the best of authors' knowledge, the non-Newtonian clear Eyring–Powell fluid and non-Newtonian nano-Eyring–Powell fluid has never been investigated together in a two layer vertical channel. This study investigates the steady fully-developed mixed convection flow in a vertical channel and the governing fluid equations in each layer of the channel are more complicated because of the presence of non-Newtonian Eyring–Powell fluid. Buoyancy force using the mathematical nano-fluid model presented by Buongiorno [19], and an outer pressure gradient is used to drive the flow. Until now, many studies have been presented which discuss the influence of different fluid parameters on Eyring–Powell fluid so, this study explicitly focuses on the effects of Brownian motion parameter, buoyancy parameter, and thermophoretic effects on this fluid and it may be considered as the extension of the problem of a two-layer flow of viscous fluid along with viscous nano-fluid investigated by Farooq and Liang [20].

In this paper, the optimal analysis homotopy method (OHAM) via Mathematica package BVPH2.0 is used to solve the governing nonlinear coupled ODEs of the non-Newtonian fluid in both layers. Residual errors and convergence control parameters for different orders of approximation are presented in the tables. Graphical results are displayed to show the influence of several interesting parameters on the fluid flow, heat, and mass transfer. Fluid behavior at the interface is also noted and discussed through the tables. A comparison between the values of the physical properties of the viscous fluid and a special case of Eyring–Powell fluid is presented through the tables.

2. Mathematical model

Mathematically the Eyring–Powell model is presented as

$$A = -pI + \Gamma \tag{1}$$

where extra stress tensor Γ is given by

$$\Gamma = \mu\, A_1 + \frac{1}{b\dot{\gamma}} \sin h^{-1}\left(\frac{1}{c}\dot{\gamma}\right)A_1 \tag{2}$$

here μ , b and c are the rheological parameters of the Eyring–Powell fluid model [21], μ is the coefficient of shear viscosity and c has the dimension of $(time)^{-1}$. We take the second order approximation of the $(\sin h)^{-1}$ function as

$$\sin h^{-1}\left(\frac{1}{c}\dot{\gamma}\right) \cong \frac{1}{c}\dot{\gamma} - \frac{1}{6}\left(\frac{1}{c}\dot{\gamma}\right)^3, \quad \left|\frac{1}{c}\dot{\gamma}\right| << 1 \tag{3}$$

and Eq. (2) takes the form

$$\Gamma = \left(\mu + \frac{1}{bc}\right)A_1 - \frac{1}{6bc^3}\dot{\gamma}^2 A_1 \tag{4}$$

where $\dot{\gamma} = \sqrt{\frac{1}{2} trA_1^2}$ and the kinematical tensor A_1 is defined as $A_1 = \nabla V + (\nabla V)^T$.

2.1. Problem formulation

Consider a two-layer vertical channel as shown in Fig. 1, also consider a steady, laminar, boundary layer, and incompressible flow between two vertical parallel plates extended in the x and z direction. l is the width of each layer and the region in the domain $0 \leqslant y \leqslant l$ is filled with clear Eyring–Powell fluid with viscosity μ_1 and density ρ_1 . The other region in the domain $-l \leqslant y \leqslant 0$ is filled with nano-Eyring–Powell fluid with viscosity μ_2 and density ρ_2. It is assumed that the pressure gradient is constant in both the regions, but the wall temperature is different for both the boundary walls of the channel. The left wall is held at temperature T_{w2} and the right wall is held at T_{w1} temperature with $T_{w1} > T_{w2}$.

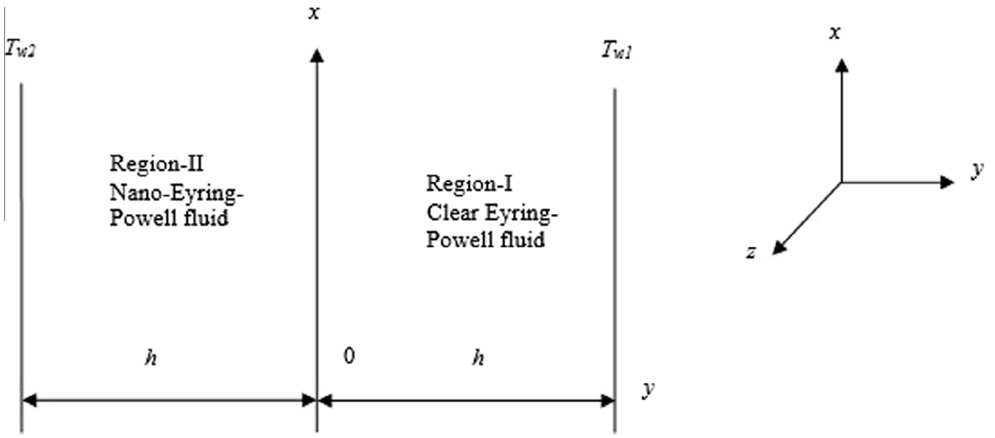

Fig. 1. Physical configuration of the problem.