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Application of new novel energy balance method to strongly nonlinear oscillator systems

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ABSTRACT

In this paper, a new novel energy balance method based on the harmonic balance method is proposed to obtain higher-order approximations of strongly nonlinear problems arising in engineering. Especially, second-order approximation is considered in this paper. Results found in this paper are compared with the exact result and other existing results. The results show that the proposed method gives better result for both small and large amplitudes of oscillation than other existing results. The method is illustrated by examples. It has been shown that the proposed method is very effective, convenient and quite accurate to nonlinear engineering problems.

1. Introduction

The study of nonlinear oscillations is important issue in engineering because many practical engineering components consist of vibrating systems that can be modeled using oscillator systems. Nonlinear oscillations are modeled by nonlinear differential equations. Many analytical approximate techniques were developed to solve these nonlinear differential equations. The traditional methods can not used to solve these nonlinear problems if no small parameter exists in equations. To overcome these shortcomings, many asymptotic techniques have been developed to solve strongly nonlinear systems such as parameter-expanding method [1,2], modified Lindstedt–Poincare method [3,4], homotopy perturbation method (HPM) [5], variational iteration method (VIM) [6–9] and energy balance method (EBM) [10,11]. He [10] obtained only first-order approximation by using energy balance method. Usually, a set of algebraic equations with complex nonlinearities appears when EBM is formulated to obtain higher-order approximations. Recently, some authors [12–14] have extended the energy balance method to obtain higher-order approximations for strongly nonlinear oscillators. Durmaz et al. [12] obtained a higher-order approximation of energy balance method based on collocation method. Durmaz and Kaya [13] used Galerkin method as weighting function to solve strongly nonlinear systems. Khan and Mirzabeigy [14] developed an improved energy balance method based on combining collocation and Galerkin–Petrov methods. In these articles [12–14], the algebraic nonlinear equations (which are not written in closed form) are solved numerically.

In this article, a new novel energy balance method based on the harmonic balance method has been presented to obtain the higher-order approximations of strongly nonlinear problems. The algebraic nonlinear equations found in this paper are solved analytically and also to be written in closed form. Generally, the second-order approximation is considered in this paper. Two examples are given to verify the accuracy and convenient of the proposed method. The results (obtained in this paper) give better results and provide high accuracy than other existing results [12–14] as compared with exact result.

2. The basis idea of He’s energy balance method

Let us consider a general form on the nonlinear problems in the following form

$$\ddot{x} + f(x) = 0, \tag{1}$$

with initial conditions

$$x(0) = A, \quad \dot{x}(0) = 0. \tag{2}$$

Its variational can be written as

$$J(x) = \int_0^{T/4} \left[-\frac{1}{2}\dot{x}^2 + F(x) \right] dt, \tag{3}$$

where $T = \frac{2\pi}{\omega}$ is a period of nonlinear oscillation and $F(x) = \int f(x)dx$.

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The Hamiltonian can be written in the following form

$$H(x) = \frac{1}{2}\dot{x}^2 + F(x) = F(A). \tag{4}$$

Eq. (4) gives the following residual

$$R(t) = \frac{1}{2}\dot{x}^2 + F(x) - F(A) = 0. \tag{5}$$

The first-order approximate solution was chosen in the following form

$$x(t) = A\cos\omega t. \tag{6}$$

Substituting Eq. (6) into Eq. (5) yields the following residual

$$R(t) = \frac{1}{2}A^2\omega^2\sin^2\omega t + F(A\cos\omega t) - F(A) = 0. \tag{7}$$

And finally collocation at $\omega t = \frac{\pi}{4}$ gives

$$\omega(A) = \frac{2}{A}\sqrt{F(A) - F\left(\frac{\sqrt{2}}{2}A\right)}. \tag{8}$$

3. New novel energy balance method

Consider the trial solution of Eq. (1) in the following form

$$x(t) = A((1-u)\cos\omega t + u\cos 3\omega t) \tag{9}$$

Eq. (9) satisfies the initial conditions given in Eq. (2).

Substituting Eq. (9) into the left-side of Eq. (5), then dividing by the factor $\sec\omega t$ and we obtain the following Fourier series expansions:

$$\left(\dot{x}^2/2 + F(x) - F(A)\right)/\sec\omega t = c_1\cos\omega t + c_3\cos 3\omega t + \cdots, \tag{10}$$

where c_1 and c_3 are calculated from the following

$$c_{2n-1} = \frac{4}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\dot{x}^2/2 + F(x) - F(A)}{\sec\omega t}\right)\cos(2n-1)\varphi\, d\varphi,\quad n = 1,2,\ldots \tag{11}$$

Substituting the right-side of Eq. (10) into the left-side of Eq. (5) and then equating the coefficients of the terms $\cos\varphi$ and $\cos 3\varphi$, we get two nonlinear algebraic equations whose solution provide the unknown frequency, ω and unknown coefficient, u in terms of amplitude A. Therefore, the determination of second-order approximation is clear.

4. Examples

4.1. Example 1

We consider a mass attached to the centre of a stretched elastic wire which is an example of a conservative nonlinear oscillatory system with an irrational elastic item. In dimensionless form, the equation of motion of this system is [1]:

$$\ddot{x} + x - \frac{\lambda x}{\sqrt{1+x^2}} = 0, \tag{12}$$

where over dots denote differentiation with respect to time t and $0 < \lambda \le 1$.

The initial conditions are

$$x(0) = A,\quad \dot{x}(0) = 0, \tag{13}$$

where A denotes the maximum amplitude.

This system oscillates between symmetric bounds $[-A, A]$, and its angular frequency and corresponding periodic solution are dependent on the amplitude A.

The variational of Eq. (12) can be written as

$$J(x) = \int_0^{T/4}\left[-\frac{1}{2}\dot{x}^2 + \frac{1}{2}x^2 - \lambda\sqrt{1+x^2}\right]dt. \tag{14}$$

Therefore, the Hamiltonian can be written in the following form

$$H(x) = \frac{1}{2}\dot{x}^2 + \frac{1}{2}x^2 - \lambda\sqrt{1+x^2} - \frac{1}{2}A^2 + \lambda\sqrt{1+A^2} = 0. \tag{15}$$

Substituting Eq. (9) into the left-side of Eq. (15), then dividing by the factor $\sec\omega t$ we obtain the following Fourier series expansions:

$$\left(\dot{x}^2/2 + \left(x^2 - A^2\right)/2 - \lambda\left(\sqrt{1+x^2} - \sqrt{1+A^2}\right)\right)/\sec\omega t = c_1\cos\omega t + c_3\cos 3\omega t + \cdots, \tag{16}$$

where

$$c_1 = -A^2 + 8\lambda\sqrt{1+A^2} + A^2\omega^2 + 4A^2\omega^2u + b + cu + \cdots,$$
$$c_3 = A^2 + 2A^2u - A^2\omega^2 + 2A^2\omega^2u + d + eu + \cdots,$$

$$b = 32\lambda((1+A^2)K(-A^2) - (1+2A^2)E(-A^2))/(3\pi A^2),$$
$$c = 128\lambda(1+8A^2)E(-A^2))/(15\pi A^4),$$

$$d = 32\lambda((A^4 - 7A^2 - 8)K(-A^2) + (8 + 3A^2 - 2A^4)E(-A^2))/(15\pi A^4),$$

$$e = 32\lambda(768 + 524A^4)K(-A^2) + (40A^6 - 108A^4)K(-A^2))/(105\pi A^6).$$

Herein, $K(m)$ and $E(m)$ are the complete elliptic integrals of the first and second kind, respectively, defined as follows [15]

$$K(m) = \int_0^{\pi/2}\frac{d\theta}{\sqrt{1 - m\cos^2\theta}}, \tag{17}$$

$$E(m) = \int_0^{\pi/2}\sqrt{1 - m\cos^2\theta}\, d\theta. \tag{18}$$

Substituting the right-side of Eq. (16) into the left-side of Eq. (15) and then equating the coefficients of the terms $\cos\varphi$ and $\cos 3\varphi$ equal to zeros, respectively, we obtain

$$-A^2 + 8\lambda\sqrt{1+A^2} + A^2\omega^2 + 4A^2\omega^2u + b + cu = 0, \tag{19}$$

$$A^2 + 2A^2u - A^2\omega^2 + 2A^2\omega^2u + d + eu = 0, \tag{20}$$

Solving Eqs. (19) and (20), we obtain the unknown coefficient, u and the second-order approximate frequency, ω as

$$u = \frac{b + d + 8\lambda\sqrt{1+A^2}}{6b - c - e - 8A^2 + 48\lambda\sqrt{1+A^2}}, \tag{21}$$

and

$$\omega(A) = \sqrt{1 - \frac{b}{A^2} - \frac{8\lambda\sqrt{1+A^2}}{A^2} + \frac{(b + d + 8\lambda\sqrt{1+A^2})(4A^2 - 4b + c - 32\lambda\sqrt{1+A^2})}{A^2(8A^2 - 6b + c + e - 48\lambda\sqrt{1+A^2})}}. \tag{22}$$

Therefore, the second-order approximation becomes

$$x(t) = A((1-u)\cos\omega t + u\cos 3\omega t), \tag{23}$$

where u_3 and ω respectively, are given in Eqs. (21) and (22).