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Hit probability of a disk shaped detector with particles with a finite range emitted by a point-like source

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ABSTRACT

An analytical analysis of the geometrical efficiency of a circular detector for particles with a finite range, emitted from a point-like source, is given. Several different cases were determined, depending on the particle range, radius of the detector and the position of the source with respect to the detector. These cases were analyzed separately and different expressions for calculating the hit probability were obtained for each of them. Results were compared with Monte Carlo calculations and good agreement was found. The problem considered here might be relevant for alpha-particle detection under specific conditions.

1. Introduction

Many authors have investigated a solid angle subtended by a circular detector at a point-like source (Gardner and Verghese, 1971; Jones, 1996; Prata, 2004; Tryka, 1997; Timus et al., 2007). Some authors treated different detector shapes (Abbas, 2006; Cook, 1980; Conway, 2010a; Favorite, 2008; Gotoh and Yagi, 1971; Green et al., 1974; Hosseini-Ashrafi and Spyrou, 1992; Prata, 2003; Wielopolski, 1984; Whitcher, 2002) and also different source geometries (Conway, 2006, 2010b; Pommé, 2007; Ruby, 1994; Yi and Jun, 1997). Knowledge of the subtended solid angle enables estimation of the geometrical efficiency of such detectors. However, even for the combination as simple as a circular detector and a point-like source, the solution of the problem is very complicated and requires strong mathematical skills. The problem becomes even more difficult if the source emitting particles with a finite range in media between the source and the detector is taken into account.

In the work presented, the hit probability was considered for a circular detector and point- like source emitting particles with a finite range in media between the source and the detector. This problem is very common when detection of alpha particles is carried out. It was treated analytically and several different cases were found. In order to simplify the problem, it was assumed that all particles emitted from the source have exactly the same range in the medium under consideration, i.e. the source is monoenergetic and range straggling is neglected. Considering this, one can imagine a sphere with radius equal to the particle range and the center located at the point-like source. No particle can reach out of this sphere and if the detector is out of it, the detection probability is equal to zero, i.e., the particle cannot be detected. In this work, a simplified assumption was made that any particle reaching the detector is registered. However, only the part of the detector lying within the sphere is exposed to particles emitted from the source. The cross section between the sphere and the detector can have different forms, depending on the radius of the detector, particle range and source position. All possible cases were analyzed and equations for calculating the probability were obtained for each of them. Some solutions contain integrals that have to be calculated numerically.

2. Method

The geometry of the problem is shown in Fig. 1. The detector is represented by a disk of radius R. The coordinate origin is located at the center of the detector and the z axis is orthogonal to it, so the detector surface belongs to the xOy plane. The source is located at an arbitrary point A with spherical coordinates (r_0, θ_0, φ_0) and it emits particles with range D in media between the source and the detector. Point A′ is the projection of point A onto the xOy plane. The problem could be simplified significantly if the

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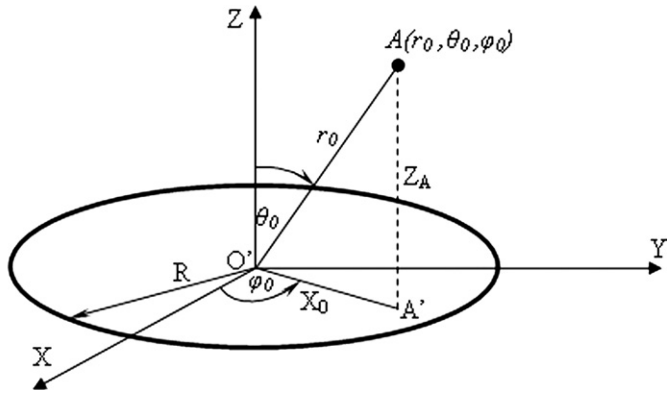

Fig. 1. Circular detector with radius R in the xOy plane; the source is at an arbitrary point A with coordinates (r_0,φ_0,θ_0) in a spherical system and (X_0,φ_0,Z_A) in a polar system.

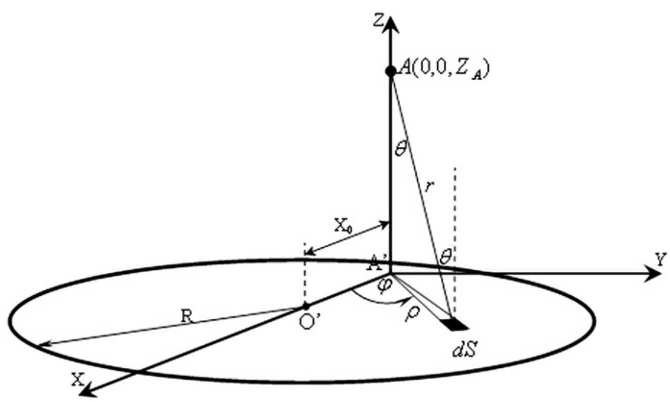

Fig. 2. Coordinate system is translated to point A′ (which is a projection of point A onto the xOy plane). The coordinate system is also rotated, so that the center of the detector is on the x axis (point O′).

coordinate origin is translated to point A′ and the x axis is chosen in such a way that the center of the detector belongs to it. This is shown in Fig. 2. The coordinates of the source in the new system are $(0,0,Z_A)$, and the coordinates of the detector center are $(X_0,0,0)$.

The following relations apply (Fig. 1):

$$X_0 = r_0 \sin\theta_0$$

$$Z_A = r_0 \cos\theta_0 \tag{1}$$

Let us now consider an elemental surface area dS of the detector, located at a distance r from the point-like source, as shown in Fig. 2. The probability of hitting the surface element dS by particles emitted from point A is given by:

$$dP = \frac{d\Omega}{4\pi} = \frac{dS\cos\theta}{4r^2\pi} \tag{2}$$

where $d\Omega$ represents the solid angle subtended by dS at the point A, and θ is the angle between r and the normal onto dS.

Based on the geometry shown in Fig. 2, this probability can also be expressed in terms of polar coordinates. The total probability of particles emitted at point A hitting the detector is obtained through integration over the specific part of the detector's surface:

$$P = \frac{Z_A}{4\pi}\int_{\varphi_{min}}^{\varphi_{max}}\int_{\rho_{min}}^{\rho_{max}} \frac{\rho\, d\rho\, d\varphi}{(\rho^2+Z_A^2)^{3/2}} \tag{3}$$

Due to the symmetry of the problem, it is enough to integrate over half of the space and then multiply the expression by a factor of two. This leads to the following equation:

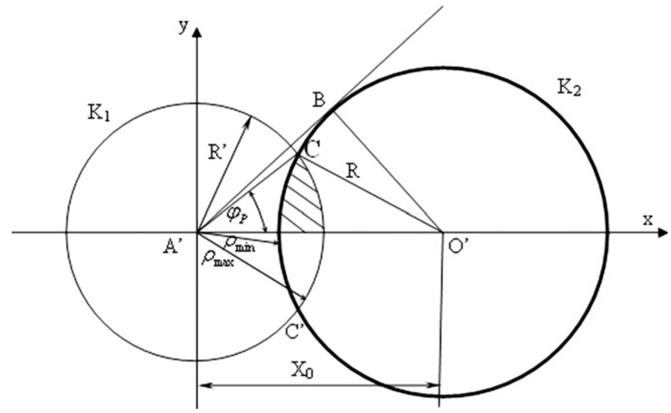

Fig. 3. Case 1. Cross section between circles K_1 and K_2 (detector). The shaded surface represents the surface of integration.

$$P = \frac{Z_A}{2\pi}\left[\int_0^{\varphi_{max}} \frac{d\varphi}{\sqrt{\rho_{min}^2+Z_A^2}} - \int_0^{\varphi_{max}} \frac{d\varphi}{\sqrt{\rho_{max}^2+Z_A^2}}\right] \tag{4}$$

The integration limits (ρ_{min}, ρ_{max} and φ_{max}) are determined by the size of the detector and the position of the source. They are obtained as the cross section between the circle $(x-X_0)^2+y^2=R^2$, which represents the detector, and the sphere of radius D, defined by the equation $x^2+y^2+(z-Z_A)^2=D^2$, with the center at point $A(0,0,Z_A)$. The cross section between the sphere and the xOy plane depends on the particle range and the location of point A with respect to the detector. One example is shown in Fig. 3. The left side circle (K_1), defined by the equation $x^2+y^2=R'^2$, represents a cross section between the xOy plane and the sphere with the center at A and radius D. R' can be obtained as $R'=\sqrt{D^2-Z_A^2}$. The right side circle (K_2) in Fig. 3 represents the detector with radius R.

In the first three cases discussed below, the coordinate origin (A') is outside the detector's surface. Using polar coordinates, one can obtain the following equation defining the detector's circle K_2 (for these three cases):

$$\rho^{\pm} = X_0\cos\varphi \pm \sqrt{R^2-X_0^2\sin^2\varphi} \tag{5}$$

For each value of the angle φ, there are two corresponding values of ρ: ρ^-, which is closer to the origin, and ρ^+, which is further away. Points (ρ^-,φ) and (ρ^+,φ) are on opposite sides of the circle K_2.

The integrals denoted by I^+ and I^- defined below will often appear in the following text:

$$\int_{\varphi_{min}}^{\varphi_{max}} \frac{d\varphi}{\sqrt{\left(X_0\cos\varphi+\sqrt{R^2-X_0^2\sin^2\varphi}\right)^2+Z_A^2}} = I^+(\varphi_{min},\varphi_{max}) \tag{6}$$

$$\int_{\varphi_{min}}^{\varphi_{max}} \frac{d\phi}{\sqrt{\left(X_0\cos\phi-\sqrt{R^2-X_0^2\sin^2\phi}\right)^2+Z_A^2}} = I^-(\varphi_{min},\varphi_{max}) \tag{7}$$

These integrals were calculated numerically.

If $X_0 > R+R'$ or $Z_A > D$, the probability is equal to 0. Both conditions mean that the detector is too far from the source and particles with range D, emitted from point A, cannot reach the detector. If these conditions are not satisfied, the probability will be larger than zero; there are several specific cases and they will be elaborated further in detail. Some conditions have been applied in order to differentiate these cases.