Physica Medica (2010) **26**, 6–16

available at www.sciencedirect.com

ScienceDirect

journal homepage: http://intl.elsevierhealth.com/journals/ejmp

ORIGINAL PAPER

Analytical theory for the fluence, planar fluence, energy fluence, planar energy fluence and absorbed dose of primary particles and their fragments in broad therapeutic light ion beams

J. Kempe*, **A. Brahme**

Division of Medical Radiation Physics, Department of Oncology-Pathology, Karolinska Institutet and Stockholm University, Stockholm, Sweden

Received 13 June 2008; received in revised form 4 February 2009; accepted 17 February 2009
Available online 3 April 2009

KEYWORDS
Light ion transport;
Fluence;
Planar fluence;
Energy fluence;
Mean energy;
Absorbed dose

Abstract The purpose of the present work is to develop analytical expressions for the depth variation of the fluence, planar fluence, the energy fluence, planar energy fluence, the mean energy and absorbed dose of primary ions and their associated fragments in tissue-like media with ranges of clinical interest. The analytical expressions of the primary ions and associated fragments take into account nuclear interactions, energy losses, range straggling and multiple scattering. The analytical models of the radiation field quantities were compared with the results of the modified Monte Carlo (MC) code SHIELD-HIT$_{+}$. The results show that the shape of the depth absorbed dose distribution of the primary particles is characterized by an increasingly steep exponential fluence decrease with depth as the charge and atomic weight increase. This is accompanied by a compensating increased energy loss towards the Bragg peak as the charge of the ion increases. These largely compensating mechanisms are the main reason that the depth absorbed dose curve of all light ions is surprisingly similar. In addition, a rather uniform dose in the plateau region is obtained since the increasing fragment production almost precisely compensates the loss of primaries. The dominating light fragments such as protons and alpha particles are characterized by longer ranges than the primaries and their depth dose curves to some extent coincide well with the depth fluence curves due to a rather slow variation of mean stopping powers. In contrast, the heavier fragments are characterized by the build up of a slowing down spectrum similar to that of the primaries but with initially slightly

* Corresponding author.
E-mail address: johanna.kempe@ki.se (J. Kempe).

doi:10.1016/j.ejmp.2009.02.003

shorter or longer ranges depending on their mass to atomic number ratio. The presented analytical theory for the light ion penetration in matter agree quite well with the MC and experimental data and may be very useful for fast analytical calculations of quantities like mean energy, fluence, energy fluence, absorbed dose, and LET.

Introduction

The therapeutic value of light ions and in particular ^{7}Li ion beams for radiation therapy, not least for paediatric patients, has recently been discussed in terms of their depth dose and LET distribution pointing out the clear separation between a low LET plateau and fragmentation region and a high LET therapeutic Bragg peak [1]. For optimal clinical application of light ions in radiation therapy, the full 3 dimensional spatial distributions in terms of radiation quality is therefore needed in order to make the best possible use of their therapeutic advantages. During the slowing down of high energy projectiles, fragments are continuously generated, with the origin either from the incoming primary projectile, the target nuclei or by fragments interactions such as by high energy secondary neutrons, protons, and α-particles and some of them may therefore be scattered almost isotropic [2—4]. These nuclear interaction processes can have a significant influence on the ion fluencies and energy fluencies with depth inside an absorber and can also influence the distribution of the absorbed dose. Analytical expressions for fast fluence and dose calculation have been proposed for ion transport in broad and narrow pencil beams [5—14]. However, there is still lack of analytical information for primary particles and their associated fragments regarding fluence, energy fluence, mean energy, and absorbed dose. The planar quantities in relation to the total fluence, energy fluence and mean energy are of interest, especially for dosimetry and absorbed dose calculations. The present study focuses on analytical expressions of radiation field quantities of broad beams of high energy light ions in matter with special emphasis on lithium and carbon ions. These analytical expressions were used to describe the basic shape of the depth absorbed dose distribution for the primary and its associated fragments. Accurate transport calculations of heavy fragments such as boron and carbon fragments generated in a high energy ^{12}C beam are of special importance for biological optimized radiation therapy, due to their high LET dose contribution in the plateau and fragmentation tail in front of and beyond the Bragg peak [1]. Furthermore, a specific developed MC code SHIELD-HIT$_+$ was used, where the fluence differential in energy and angle was calculated, to derive the fundamental radiation field quantities such as fluence, planar fluence, energy fluence, planar energy fluence, mean energy, and angular distribution of the primary and all fragments in high energy ion beams. These quantities were used for comparison with the presently developed analytical transport theory for primary ions and their generated fragments. Here the recently derived energy—range relations [15] have been used to develop quite accurate analytical expressions for the transport of the primary particles and their associated light and heavy fragments.

Theory and methods

Fluence, vectorial fluence and planar fluence of particles

The double differential ion fluence in energy, E, and angle, $\hat{\Omega}$, at a point defined by the coordinate vector $\boldsymbol{r}$ is generally written as $\Phi_{E,\Omega}(\boldsymbol{r})$ and if it is known for all the primaries and fragments it fully describes the ion transport. For clarity, we use the basic ICRU notations [16,17]. Based on this double differential ion fluence all other time independent fluence concepts can be derived, cf. Fig. 1. The fluence and the vectorial fluence respectively are obtained from

$$\Phi(\boldsymbol{r}) = \iint \Phi_{E,\Omega}(\boldsymbol{r}) \mathrm{d}E\, \mathrm{d}\Omega, \tag{1}$$

$$\boldsymbol{\Phi}(\boldsymbol{r}) = \iint \hat{\boldsymbol{\Omega}} \Phi_{E,\Omega}(\boldsymbol{r}) \mathrm{d}E\, \mathrm{d}\Omega = \frac{\iint \hat{\boldsymbol{\Omega}} \Phi_{E,\Omega}(\boldsymbol{r}) \mathrm{d}E\, \mathrm{d}\Omega}{\iint \Phi_{E,\Omega}(\boldsymbol{r}) \mathrm{d}E\, \mathrm{d}\Omega} \Phi(\boldsymbol{r}) = \overline{\boldsymbol{\Omega}}^{\Phi}(\boldsymbol{r}) \Phi(\boldsymbol{r}), \tag{2}$$

where $\overline{\boldsymbol{\Omega}}^{\Phi}(\boldsymbol{r})$ corresponds to the fluence weighted mean value of the direction of particle motion as defined by the last part of Eq. (2). The planar fluence, $\Phi^p(\boldsymbol{r}, \hat{\omega})$, is defined as the number of particles which traverse a plane surface with the normal in the direction of a given unit vector $\hat{\omega}$, per unit area of that surface [16]. The planar component of the vectorial fluence in the direction of a perpendicular incident to the surface on a uniform media with $\hat{\omega} = \hat{\boldsymbol{z}}$ defines the planar fluence in the forward direction according to:

$$\Phi^p(\boldsymbol{r}, \hat{\boldsymbol{z}}) = \iint \hat{\boldsymbol{z}} \cdot \hat{\boldsymbol{\Omega}} \Phi(\boldsymbol{r})_{E,\Omega} \mathrm{d}E\, \mathrm{d}\Omega = \frac{\iint \cos\Theta \Phi(\boldsymbol{r})_{E,\Omega} \mathrm{d}E\, \mathrm{d}\Omega}{\iint \Phi(\boldsymbol{r})_{E,\Omega} \mathrm{d}E\, \mathrm{d}\Omega} \Phi(\boldsymbol{r}) = \overline{\cos\Theta}^{\Phi} \Phi(\boldsymbol{r}), \tag{3}$$

where Θ is the angle between $\hat{\boldsymbol{z}}$ and $\hat{\boldsymbol{\Omega}}$, i.e. the direction of propagation relative to the forward direction $\hat{\boldsymbol{z}}$ of each particle, and $\overline{\cos\Theta}^{\Phi}$ is the fluence weighted mean direction cosine for all particles as defined by the last half of Eq. (3) similar to $\overline{\boldsymbol{\Omega}}^{\Phi}$ in Eq. (2), cf. Fig. 1.

Using the vectorial fluence, Eq. (2), the planar fluence, Eq. (3), in an arbitrary direction $\hat{\omega}$ can be written in a more general form as

$$\Phi^p(\boldsymbol{r}, \hat{\omega}) = \hat{\omega} \cdot \boldsymbol{\Phi}(\boldsymbol{r}) = \hat{\omega} \cdot \overline{\boldsymbol{\Omega}}^{\Phi} \Phi(\boldsymbol{r}) = \left|\overline{\boldsymbol{\Omega}}^{\Phi}\right| \cos\theta \cdot \Phi(\boldsymbol{r}), \tag{4}$$

where θ is the angle between the normal $\hat{\omega}$ of the reference surface and the mean direction $\overline{\boldsymbol{\Omega}}^{\Phi}$ of the ions in the beam. In beams of incident perpendicularly to a uniform phantom surface with normal in direction $\hat{\boldsymbol{z}}$ and azimuthally symmetric scattering, $\hat{\boldsymbol{\Omega}}^{\Phi} \equiv \hat{\boldsymbol{z}}$, $\hat{\boldsymbol{\Omega}}^{\Phi}$ is a unit vector in the mean direction of the particles given by $\hat{\boldsymbol{\Omega}}^{\Phi} = \overline{\boldsymbol{\Omega}}^{\Phi} / |\overline{\boldsymbol{\Omega}}^{\Phi}|$.