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Parametric X-rays along the velocity direction of an emitting particle under conditions of the Cherenkov effect

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Abstract

Peculiarities in the parametric X-rays along the velocity direction of a relativistic electron emitting in a regime where the Cherenkov effect is important are considered. It is shown that the emission yield can be increased substantially due to the modification of the anomalous photoabsorption.

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1. Introduction

The theory of parametric X-rays (PXR) from fast-moving charged particles in a periodic medium predicts the existence of a PXR peak not only in the direction of Bragg scattering but also along the velocity direction of the emitting particle (Garibian and Yang, 1971; Baryshevsky and Feranchuk, 1971; Baryshevsky, 1997; Kubankin et al., 2003). Such an additional X-ray peak (the forward PXR) is of great interest because up to now the question concerning its nature as a Cherenkov–like or scattering process, is still open.

It should be noted that the forward PXR from crystals has only recently been the subject of investigations (see, e.g. the experimental observations of Kube et al., 2001; Aleinik et al., 2004). Because of this, only the Bragg emission process in the hard X-ray range, where the average dielectric susceptibility of the target $\chi_0(\omega)$ is negative, has been studied. Forward PXR in the soft X-ray range, where χ_0 can be positive in the vicinity of a photoabsorption edge of the target material, is considered in our paper. Macroscopic periodic nanostructures are best suited for the realization of such an emission process. The aim of this work is to describe the peculiarities in forward PXR appearing under conditions of the Cherenkov effect. Among other things, substantial enhancement of the PXR intensity is predicted in our paper.

2. Statement of the problem

Let us consider the structure of the electromagnetic field excited by a relativistic electron emitted from a target with a periodically changing dielectric susceptibility $\chi(\omega, \mathbf{r}) = \chi_0(\omega) + \sum_{\mathbf{g}\neq\mathbf{0}}\chi_{\mathbf{g}}(\omega)e^{i\mathbf{g}\cdot\mathbf{r}}$, in which the $\mathbf{g}$'s

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are reciprocal lattice vectors. The geometry of the emission process is presented in Fig. 1. A simple model of a semi-infinite target is used to describe the emission characteristics, so that the thickness of the target is larger than an absorption length. Starting from the general Maxwell equations for the Fourier-transform of an electric field $\mathbf{E}_{\omega\mathbf{k}} = (2\pi)^{-4}\int \mathrm{d}t\,\mathrm{d}^3r\mathbf{E}(\mathbf{r},t)\mathrm{e}^{\mathrm{i}\omega t-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}$

$$(k^2-\omega^2(1+\chi_0))\mathbf{E}_{\omega\mathbf{k}} - \mathbf{k}(\mathbf{k}\cdot\mathbf{E}_{\omega\mathbf{k}}) - \omega^2\sum_{\mathbf{g}\neq\mathbf{0}}\chi_{-\mathbf{g}}(\omega)\mathbf{E}_{\omega\mathbf{k}+\mathbf{g}} = \frac{\mathrm{i}\omega e}{2\pi^2}\mathbf{V}\delta(\omega-\mathbf{k}\cdot\mathbf{V}), \tag{1}$$

we determine the excited field within the framework of the two-wave approximation of dynamical diffraction theory (Pinsker, 1984) often and with great success employed in theoretical studies of PXR. The quantity $\mathbf{V}=(V_x,V_y,V_z)$ in Eq. (1) is the velocity of an emitting electron. Reasoning that

$$\mathbf{E}_{\omega\mathbf{k}} \simeq \sum_{\lambda=1}^{2}\mathbf{e}_{\lambda 0}E_{\lambda 0}, \quad \mathbf{k}\cdot\mathbf{e}_{\lambda 0}=0,$$

$$\mathbf{E}_{\omega\mathbf{k}+\mathbf{g}} \simeq \sum_{\lambda=1}^{2}\mathbf{e}_{\lambda g}E_{\lambda\mathbf{g}}, \quad (\mathbf{k}+\mathbf{g})\cdot\mathbf{e}_{\lambda g}=0, \tag{2}$$

where $\mathbf{e}_{10}=\mathbf{e}_{1\mathbf{g}}\sim\mathbf{k}\times\mathbf{g}$, $\mathbf{e}_{20}\sim\mathbf{k}\times\mathbf{e}_{10}$, $\mathbf{e}_{2\mathbf{g}}\sim(\mathbf{k}+\mathbf{g})\times\mathbf{e}_{10}$ are the polarization vectors, one can reduce (1) to the simple relationships

$$(k^2-\omega^2(1+\chi_0))\mathbf{E}_{\lambda 0} - \omega^2\chi_{-\mathbf{g}}\alpha_\lambda E_{\lambda\mathbf{g}} = \frac{\mathrm{i}\omega e}{2\pi^2}\mathbf{e}_{\lambda 0}\cdot\mathbf{V}\delta(\omega-\mathbf{k}\cdot\mathbf{V}), \tag{3}$$

$$((\mathbf{k}+\mathbf{g})^2-\omega^2(1+\chi_0))\mathbf{E}_{\lambda\mathbf{g}} - \omega^2\chi_{\mathbf{g}}\alpha_\lambda E_{\lambda 0} = 0$$

well known in PXR theory. Here, $\alpha_1=1$, $\alpha_2=(\mathbf{k}\cdot(\mathbf{k}+\mathbf{g}))/k|\mathbf{k}+\mathbf{g}|$.

In the case of a Laue scattering geometry and a semi-infinite target the solution of Eq. (3) can be determined

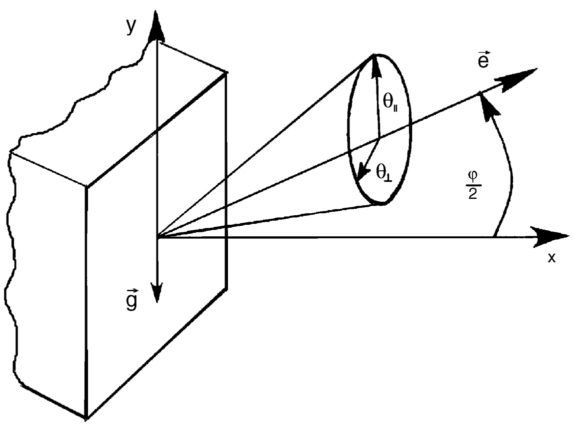

Fig. 1. The geometry of the emission process. $\mathbf{g}$ is the reciprocal lattice vector, $\mathbf{e}$ is the axis of the X-ray detector, $\varphi/2$ is the orientation angle, $\theta_\parallel$ and $\theta_\perp$ are the observation angles.

without regard for the free waves in the target. Then the components $E_{\lambda 0}$ are given by

$$E_{\lambda 0} = \frac{\mathrm{i}\omega e}{2\pi^2}\frac{((\mathbf{k}+\mathbf{g})^2-\omega^2(1+\chi_0))(\mathbf{e}_{\lambda 0}\cdot\mathbf{V})\delta(\omega-\mathbf{k}\cdot\mathbf{V})}{(k^2-\omega^2(1+\chi_0))((\mathbf{k}+\mathbf{g})^2-\omega^2(1+\chi_0))-\omega^4\chi_g\chi_{-g}\alpha_\lambda^2}. \tag{4}$$

The solution in the vacuum behind the target follows from (3) with $\chi_0=\chi_g=\chi_{-g}=0$

$$E_{\lambda 0}^{\mathrm{Vac}} = \frac{\mathrm{i}\omega e}{2\pi^2}\frac{(\mathbf{e}_{\lambda 0}\cdot\mathbf{V})\delta(\omega-\mathbf{k}\cdot\mathbf{V})}{(k^2-\omega^2)} + a_{\lambda\mathbf{k}_\parallel}\delta\left(k_x-\sqrt{\omega^2-k_\parallel^2}\right), \tag{5}$$

where $\mathbf{k}_\parallel=\mathbf{k}_y+\mathbf{k}_z$.

Determining the coefficient $a_{\lambda\mathbf{k}_\parallel}$ by boundary conditions at the surface of the target and calculating the Fourier-integral $E_\lambda^{\mathrm{Rad}}=\int\mathrm{d}^3k\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}a_{\lambda\mathbf{k}_\parallel}\delta(k_x-\sqrt{\omega^2-k_\parallel^2})$ in the wave-zone (at the distance from an emitting electron much greater than the emission formation length $l_{\mathrm{coh}}\approx(2/\omega)(\gamma^{-2}-\chi_0+\theta^2)^{-1}$) by the stationary phase method one can obtain the following formula for the emission amplitude A_λ

$$E_\lambda^{\mathrm{Rad}} \to A_\lambda\frac{\mathrm{e}^{\mathrm{i}\omega r}}{r},$$

$$A_\lambda = -2\pi\mathrm{i}\omega n_x a_{\lambda\omega\mathbf{n}_\parallel} = \frac{e}{\pi}\frac{n_x}{V_x}(\mathbf{e}_{\lambda 0}\cdot\mathbf{V})\left[\frac{D+(2gn_y/\omega)(g/2\omega n_y-1)}{D\cdot(D+2(gn_y/\omega)(g/2\omega n_y-1))-\chi_g\chi_{-g}\alpha_\lambda^2}-\frac{1}{D+\chi_0}\right], \tag{6}$$

$$D=\frac{1}{V_x^2}(1-n_yV_y)^2-n_x^2-\chi_0.$$

Here, $\mathbf{n}=(n_x,n_y,n_z)$ is the unit vector denoting the direction of the emitted photon propagation. Using angular variables $\theta_\parallel$ and $\theta_\perp$ and φ introduced in Fig. 1 one can simplify (6) and represent A_λ in the final form

$$A_\lambda = \frac{e}{2\pi}\theta_\lambda\left[\left(1+\frac{\Delta}{\kappa_\lambda}\right)\frac{1}{\Delta_0+\Delta-\kappa_\lambda} + \left(1-\frac{\Delta}{\kappa_\lambda}\right)\frac{1}{\Delta_0+\Delta+\kappa_\lambda}-\frac{2}{\gamma^{-2}+\theta^2}\right] \tag{7}$$

in which

$$\Delta_0=\gamma^{-2}-\chi_0+\theta^2, \quad \kappa_\lambda=\sqrt{\Delta^2+\chi_g\chi_{-g}\alpha_\lambda^2},$$

$$\Delta=2\sin^2\left(\frac{\varphi}{2}\right)\left(\frac{\omega_{\mathrm{B}}'}{\omega}-1\right)$$

and where the first term is inversely related to the emission formation length l_{coh}, the second term describes an influence of dynamical diffraction effects, the last term is the so-called Bragg resonance defect, describing the deviation of the system of primary virtual photon and diffracted PXR wave from the condition of