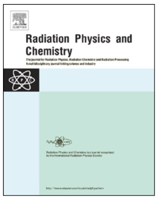
Radiation Physics and Chemistry 106 (2015) 204–212

Contents lists available at ScienceDirect

Radiation Physics and Chemistry

journal homepage: www.elsevier.com/locate/radphyschem



Power output and efficiency of beta-emitting microspheres

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HIGHLIGHTS

- Range-energy relationship for the beta particles in yttrium-90 is calculated.
- Formalism for the semi-analytical calculation of self-absorption coefficients.
- Energy-dependent self-absorption coefficient calculated for yttrium-90.
- Flux rate of beta particles from a self-attenuating radioactive sphere is shown.
- The efficiency of beta particle emitting radioactive microspheres is calculated.

ARTICLE INFO

Article history:
Received 6 February 2014
Accepted 30 July 2014
Available online 6 August 2014

Keywords:
Self-absorption
Beta-decay
Dosimetry
Y-90
Microspheres
Mass attenuation

ABSTRACT

Current standard methods to calculate the dose of radiation emitted during medical applications by beta-minus emitting microspheres rely on an over-simplistic formalism. This formalism is a function of the average activity of the radioisotope used and the physiological dimensions of the patient only. It neglects the variation in energy of the emitted beta particle due to self-attenuation, or self-absorption, effects related to the finite size of the sphere. Here it is assumed the sphere is comprised of a pure radioisotope with beta particles being emitted isotropically throughout the material. The full initial possible kinetic energy distribution of a beta particle is taken into account as well as the energy losses due to scattering by other atoms in the microsphere and bremsstrahlung radiation. By combining Longmire's theory of the mean forward range of charged particles and the Rayleigh distribution to take into account the statistical nature of scattering and energy straggling, the linear attenuation, or self-absorption, coefficient for beta-emitting radioisotopes has been deduced. By analogy with gamma radiation transport in spheres, this result was used to calculate the rate of energy emitted by a beta-emitting microsphere and its efficiency. Comparisons to standard point dose kernel formulations generated using Monte Carlo data show the efficacy of the proposed method. Yttrium-90 is used as a specific example throughout, as a medically significant radioisotope, frequently used in radiation therapy for treating cancer.

1. Introduction

Beta-minus emitting radioactive microspheres are increasingly used in medical applications such as PET/SPECT imaging (Pasciak et al., 2014; Elschot et al., 2013; D'Arienzo et al., 2012; Kao et al., 2012) and radiation therapy (Houle et al., 1989; Martin et al., 2012; Salem and Thurston, 2006; Kennedy et al., 2007; Dadachova et al., 2002; Shukla et al., 2012; Kim and Burgess, 2002; Chanda et al., 2010) as they often offer advantages over alternative methods such as chemoembolization and external beam radiation (Hoffmann et al., 2011; Gates, 2007). For instance, in radioembolization (Martin et al., 2012; Kennedy et al., 2007; Vente et al., 2009) or Selective Internal Radiation Therapy (SIRT) (Stubbs and Cannan, 2002), radioactive microspheres are delivered to a tumour through the bloodstream via a catheter prior to an angiogram. The particles lodge in the tumour and emit radiation that kills the tumour. This method of cancer treatment is commonly used in conjunction with more well-established treatments such as chemotherapy. While this method offers advantages over external beam radiation, there is still a danger of complication due to extrahepatic deposition of the radioactive material (Crowder et al., 2009; Domínguez-Gadea and Cerezo, 2011; Riaz et al., 2009; Aydarous, 2008). To minimise the potential of further injury, the dose of radiation delivered to the patient needs to be well controlled (Domínguez-Gadea and Cerezo, 2011; Garin et al., 2011). However, the absorbed dose, as

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http://dx.doi.org/10.1016/j.radphyschem.2014.07.019
0969-806X/

described by a widely used formalism (Salem and Thurston, 2006; Jones and Mallard, 1963; Mo et al., 2005), is a function of the expected activity and pertinent physiological dimensions only, which does not include the variation in energy of the emitted electron due to self-attenuation effects related to the finite size of the particle.

There are many types of radionuclides used in medicine and generally are used as spheres of various sizes and compositions (Dezarn, 2008; Brouwers et al., 2004; DeNardo et al., 2004; Miller et al., 2005; Bouchat et al., 2010; Häfeli, 2002). The energy of the electrons emitted by these radioactive microspheres can vary over several orders of magnitude and for lower energy electrons, energy loss within the microsphere can be significant. This effect can be compounded if the microspheres cluster during radio-immunotherapy (Campbell et al., 2000; Howell et al., 1989).

The electrons lose energy as they travel through the microsphere due to interactions and collisions with atoms (Krane, 1988; Carron, 2007). During these interactions, atoms can absorb energy, slowing down the electron and deflecting it from its original trajectory. This is self-attenuation. The deflection of the electrons trajectory is mainly due to elastic scattering, inelastic scattering, ionization and Bremsstrahlung radiation. There are other scattering processes and loss mechanisms such as Moller scattering etc. but these are relatively unimportant (Krane, 1988). Elastic scattering occurs due to Coulomb interactions between the electron and an atomic nucleus screened by the atomic electrons. Elastic scattering does not result in a loss of energy in the electron and leaves the atom undisturbed. During an inelastic interaction, or excitation, the incident electron interacts with atomic electrons. This can raise the atomic electron into an excited state while the original continues with reduced energy. During ionization, the incident electron knocks out one of the atomic electrons, thus ionising the atoms. Bremsstrahlung radiation can be emitted when the electron is briefly accelerated as it is being deflected. The accompanying energy loss in the form of photons is possibly significant only at relativistic energies, i.e. >1 MeV (Carron, 2007). As the emitted electron passes through matter, it will interact with many atoms along its path. The electrons trajectory will therefore deviate from its original straight-line path, losing energy due to excitation and ionization and given enough material, eventually come to rest. Therefore, the true energy emitted by a radioactive particle is not just a function of its activity but of its size as well (Ahlen, 1980). This is captured within a self-attenuation coefficient which is characteristic of the radioactive material. Measurement of this coefficient is non-trivial as many factors need to be taken into account. To this end, a number of researchers are still trying to generate accurate empirical relationships for the related mass attenuation coefficient albeit for non-radioactive absorbers (Demir and Turşucu, 2012; Ermis and Celiktas, 2012).

Calculations on the energy lost by beta particles have largely been focussed on aqueous media because of the important medical implications (Berger, 1973; Bardies and Chatal, 1994; Pérez et al., 2011). These models are generally formulated using either solutions of the transport equations Spencer, 1959; Prestwich et al.,1989, Monte Carlo calculations (Pérez et al., 2011; Papadimitroulas et al., 2012; Divoli et al., 2009), or convolutions of tabulated data from experimental sources such as thin foil (Berger, 1963) or PET/SPECT measurements (Syme et al., 2003; Sgouros and Hobbs, 2014). Each of these methods have their advantages and limitations. Solving the transport equations, as is used here for instance, is a fast and robust method but without the previously mentioned mass attenuation coefficients and analytical solutions for the spherical geometry, requires significant numerical calculations (Prestwich et al., 1989). Similarly, Monte Carlo methods have been very successfully used in dosimetry, but require substantial computational effort to generate sufficient data sets (Papadimitroulas et al., 2012; Divoli et al., 2009). Fortunately codes such as FLUKA (Botta et al., 2011) and Geant4 (Freudenberg et al., 2011) are becoming increasingly robust and sophisticated and are being adapted for this purpose. Monte Carlo calculations are frequently used to generate 'point dose kernels' which are non-physical functions fitted to the numerical data to represent the energy absorbed by various, usually aqueous, media upon exposure to a point beta particle source. These kernels could then be used to numerically calculate the dose due to exposure to multiple or distributed sources. One of the most frequently cited references was by Berger (1973) who used the ETRAN code to simulate monoenergetic electrons and produced dose point kernels that took into account multiple Coulomb scattering and energy transport by bremsstrahlung production. This work was extended by Prestwich et al. (1989) who calculated the pose point kernels for six radionuclides of interest in nuclear medicine (Y-90, P-32, Cu-67, I-131, Re-186 and Re-188) considering only beta particle emission but took into account the full energy spectrum. More recently, Cross et al. (1992) proposed empirical calculations of doses delivered by point sources at a distance. Most of the codes used to generate the point source data are limited by the validity of the multiple diffusion theory at low energies and therefore are rarely applicable below 1–20 keV (Zaidi and Sgouros, 2010). These point dose kernels are rarely amenable to analytical calculation unless they are in a convenient form Sarfaraz and Wessels (1999), Leichner (1994).

In this paper, the power emitted by a radioactive sphere, and hence its activity, is calculated. Expressions developed by Fermi (1950, 1934) for the energy distribution of the electron immediately after decay including correction factors that include the effect of Coulomb interactions are used to determine the initial activity. The deviation of the trajectory of the electrons caused by multiple scattering and the energy loss due to ionization and excitation are calculated assuming a continuum model whereby scattering processes are continuous and not discreet. This model is used to calculate the change in the energy distribution of the electrons as they travel through the material and by fitting an exponential model to this data, the self-absorption coefficient is determined. By analogy with an analytical solution recently developed for a spherical gamma emitter by Atkinson and Brezovich (2006), the total power output and efficiency of a beta-emitting microsphere is calculated. While the theory presented can be applied to any beta emitting sphere for any application, the specific case of a sphere composed of pure Yttrium-90, or Y-90, will be considered as an example. This is because Y-90 is a medically significant isotope of Yttrium frequently used in radiation therapy for treating cancer. Results are compared to point dose kernel formulations generated by fitting functional forms to tabulated Monte Carlo data which show the efficacy of the proposed method.

2. Theory

During beta-minus decay, as considered here, a neutron converts to a proton causing two particles to be ejected from the nucleus: the electron and the antineutrino. In the case of Y-90, which has an atomic number, Z, of 39, the result is an isotope of Zirconium, Z-90 with an atomic number, Z', of 40. The available kinetic energy released by the reaction is divided between the antineutrino and the electron. Therefore, the kinetic energy of the emitted electron, T, can take any value less than the maximum kinetic energy available, Q, given by Krane (1988):

$$Q = M_1c^2 - M_2c^2 - m_ec^2 - m_{\bar{\nu}}c^2 \tag{1}$$

M_1 is the nuclear mass of the original parent nucleus, M_2 is the nuclear mass of the daughter nucleus after decay and m_e is the