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Effects of misaligned electron beam on inverse free electron laser acceleration ☆

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HIGHLIGHTS

- Electromagnetic and magnetostatic wiggler IFEL acceleration scheme is presented.
- Electromagnetic wiggler has higher accelerating gradient at lower saturation level.
- Electromagnetic wiggler has more tolerance for angle than magnetostatic wiggler.

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ABSTRACT

In this paper, we discuss the effects of misaligned electron beam on an inverse free electron laser with both an electromagnetic wave wiggler and magnetostatic wiggler acceleration scheme. It is shown analytically that electromagnetic wiggler IFEL energy gain distance is substantially smaller when compared to the standard IFEL i.e. with a magnetostatic wiggler. The analysis further explains a better tolerance of the electromagnetic wiggler IFEL with respect to the misaligned electron beam in comparison to a magnetostatic wiggler IFEL scheme.

1. Introduction

There exists an interest in inverse free electron lasers for laser accelerations with higher accelerating gradients. In an IFEL accelerator, there is a transfer of energy between the laser and the electron beam in presence of an undulator magnetic field. In IFEL, relativistic electrons are moving through an undulator magnetic field feels a small transverse velocity in a direction parallel to the electric field vector of a co-propagating electromagnetic wave and helps energy exchange between the electron and the wave. IFELs have been demonstrated for accelerating electrons to the energies of the order of GeV and do not suffer the limitation of the conventional microwave accelerating structure. In the 1970s (Palmer, 1972), the basic principle of IFEL was proposed with a helical wiggler. Later (Courant et al., 1985) it was shown that the IFEL can accelerate electrons to few hundred GeV with an average acceleration rate of the order of 200 MeV/m and the effects of synchrotron radiation losses were explained for both linear and helical magnetostatic undulator magnetic field.

The IFEL with a square wave wiggler scheme (Parsa and Pato, 1997) was proposed where it was shown to exhibit an energy gain of two in comparison to the conventional IFEL with sinusoidal field wiggler. Conceptual design of a 300 GeV accelerator based on the IFEL process was represented (Pellegrini et al., 1983). The study of IFEL on potential mode of electron acceleration has been pursued by Brookhaven National Laboratory (Fisher et al., 1994). The theory of IFEL beat wave accelerator was proposed. In this scheme (Bobin 1985; Cai and Bhattacharjee, 1990) the presence of plasma, under suitable conditions enhance the accelerating electric field in the IFEL. In the scheme, the beat wave generated by a laser and an undulator couple to plasma oscillation generated by the electron beam in the plasma. A chirped pulse IFEL vacuum accelerator for high gradient laser acceleration in vacuum was proposed (Hartmann et al.). An undulator with non adiabatic tapering for IFEL was reported (Varfolomeev et al., 2002). A taper in both the magnetic field and the undulator period are reported to provide synchronicity of the laser beam interaction with a captured electron bunch along the whole undulator.

In recent years there exists interest in IFEL using compact helical undulator (Musumeci et al., 2005; Duris et al., 2011; Anderson et al., 2011; Tremainc et al., 2011). The reported work on two-stage optical IFEL gives a new promise (Dunning et al., 2013) for high gradient IFELs. Electron acceleration in the IFEL with a helical wiggler in the presence of ion-channel guiding magnetic

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field has been investigated. The effects of tapering wiggler amplitude and axial magnetic field are calculated for the electron acceleration (Khazaeinezhad and Esmaeilzadeh, 2012).

In this paper we reconsider the theory of IFEL with an electromagnetic wiggler (Lu et al., 1992; Esmaeilzadeh and Taghavi, 2010; Freund et al., 1986; Olumi et al., 2011; Esmaeilzadeh et al., 2006). The wiggler is a circularly polarized electromagnetic wiggler. The study is aimed at comparing the electromagnetic wiggler IFEL with IFEL with a helical magnetostatic wiggler field. It is shown that the electromagnetic wiggler IFEL provides smaller energy gain distance of the electron as compared to a helical magnetostatic wiggler.

2. Theory of IFEL with imperfect helical trajectory

We consider an electromagnetic wiggler who‘s magnetic and electric fields are expressed as (Lu et al., 1992; Esmaeilzadeh and Taghavi, 2010; Freund et al., 1986; Olumi et al., 2011; Esmaeilzadeh et al., 2006)

$$B_w(z,t)=B_w\left[\cos(k_wz+\omega_wt),\ \sin(k_wz+\omega_wt),0\right]$$
$$E_w(z,t)=\frac{\omega_wB_w}{k_wc}\left[\sin(k_wz+\omega_wt),\ \cos(k_wz+\omega_wt),0\right] \tag{1}$$

where B_w is the peak magnetic field of the wiggler of the wavelength $\lambda_w=2\pi/k_w$. k_w and ω_w are the wave number and the frequency of the electromagnetic wiggler, respectively. The electromagnetic wave propagating along the wiggler is described by,

$$\vec{E}_L=[E_0\ \sin\ \psi,E_0\ \cos\ \psi,0]$$
$$\vec{B}_L=[-E_0\ \cos\ \psi,E_0\ \sin\ \psi,0] \tag{2}$$

where $\psi=n(kz-\omega t)$ and $k=2\pi/\lambda$, λ is the wavelength of the laser. The equation describing the motion of the electrons in the IFEL can be derived from the Lorentz equation of motion,

$$\frac{d(\vec{\beta})}{dt}=\frac{e}{m_ec\gamma}\left[(\vec{E}_L+\vec{E}_w)+\vec{\beta}\times\left(\vec{B}_L+\vec{B}_w\right)\right] \tag{3}$$

In components Eq. (3) reads,

$$\frac{d\vec{\beta}_x}{dt}=\frac{eE_0}{m_ec\gamma}(1-\beta_z)\sin\ \psi-\frac{eB_w}{m_ec\gamma}\left(\frac{\omega_w}{k_wc}+\beta_z\right)\sin(k_wz+\omega_wt)$$
$$\frac{d\vec{\beta}_x}{dt}=\frac{eE_0}{m_ec\gamma}(1-\beta_z)\cos\ \psi-\frac{eB_w}{m_ec\gamma}\left(\frac{\omega_w}{k_wc}+\beta_z\right)\cos(k_wz+\omega_wt) \tag{4}$$

The electron velocity is given by,

$$\beta_x=\frac{K}{\gamma}\cos(k_wz+\omega_wt)+\frac{K_L}{\gamma}\cos\ \psi \tag{5a}$$

$$\beta_y=\frac{K}{\gamma}\sin(k_wz+\omega_wt)-\frac{K_L}{\gamma}\sin\ \psi \tag{5b}$$

where the wiggler and electromagnetic wave parameter is defined as

$$K=\frac{eB_w\lambda_w}{2\pi m_ec^2},\quad K_L=\frac{1}{n}\frac{eE_0\lambda}{2\pi m_ec^2}$$

let $k_wv_z+\omega_w=\Omega$ then the electron velocity and trajectories are

$$\beta_x=\frac{K}{\gamma}\cos\ \Omega t+\frac{K_L}{\gamma}\cos\ \psi \tag{6a}$$

$$\beta_y=\frac{K}{\gamma}\sin\ \Omega t-\frac{K_L}{\gamma}\sin\ \psi \tag{6b}$$

$$x=c\frac{K}{\gamma\Omega}\sin\ \Omega t+c\frac{K_L}{\gamma(\beta_z-1)\omega}\sin\ \psi \tag{7a}$$

$$y=-c\frac{K}{\gamma\Omega}\cos\ \Omega t+c\frac{K_L}{\gamma(\beta_z-1)\omega}\cos\ \psi \tag{7b}$$

For a misaligned electron beam (Colson et al., 1985) we write Eqs. (6a) and (6b) as

$$\beta_x=\frac{K}{\gamma}\cos\ \Omega t+\frac{K_L}{\gamma}\cos\ \psi+\theta \tag{8a}$$

$$\beta_y=\frac{K}{\gamma}\sin\ \Omega t-\frac{K_L}{\gamma}\sin\ \psi \tag{8b}$$

Substituting β_x and β_y from Eqs. (8a) and (8b) in the relation $\beta^2=1-(1/\gamma^2)$ we get the longitudinal velocity as

$$\beta_z=\beta^*-\frac{K\theta}{\gamma}\cos(\Omega t) \tag{9}$$

where

$$\beta^*=1-\frac{1}{2\gamma^2}\left[1+K^2+\gamma^2\theta^2\right]$$

The longitudinal co-ordinate can be evaluated by integrating the above expression to get

$$z=\beta^*ct+c\frac{K\theta}{\gamma\Omega}\sin(\Omega t) \tag{10}$$

The electron longitudinal co-ordinate can be expressed from Eq. (10) as $z=\bar{z}+\Delta z$, where $\bar{z}=\beta^*ct$with

$$\Delta z=c\frac{K\theta}{\gamma\Omega}\sin\ \Omega t \tag{11}$$

The change in electron energy is given by

$$\frac{d\gamma}{dt}=\frac{e}{m_ec}\vec{E}.\vec{\beta} \tag{12}$$

Using Eqs. (8a),(8b) and (11), the change in electron energy is given by

$$\frac{d\gamma}{dz}=A\left[\frac{K}{\gamma}\sin(\psi+\Omega t)+\theta\ \sin(\psi)\right] \tag{13}$$

where

$$A=\frac{eE_0}{m_ec^2}$$

when Eq. (12) is averaged over a discrete number of wiggler periods, the fast oscillating term ψ become very small or zero. The phase term can be written in a simplified form as

$$\psi+\Omega t=n\xi+n\chi\ \sin\ \Omega t-(n-1)\Omega t \tag{14}$$

where

$$\xi=k\bar{z}-\omega t+\Omega t,\quad \chi=c\frac{Kk\theta}{\gamma\Omega} \tag{15}$$

Using Eqs. (14) and (13) we get,

$$\frac{d\gamma}{dz}=\frac{AK}{2i\gamma}\left[\begin{array}{c}\exp(i\{n\xi-(n-1)\Omega t+n\chi\ \sin\ \Omega t\})\\ -\exp(-i\{n\xi-(n-1)\Omega t+n\chi\ \sin\ \Omega t\})\end{array}\right] \tag{16}$$

where we have used the identity

$$\exp\{-in\chi\ \sin\ \Omega t\}=\sum_m J_m(n\chi)\exp(-im\Omega t)$$

$J_m(x)$ is the cylindrical Bessel function. Eq. (16) is further simplified to

$$\frac{d\gamma}{dz}=\frac{AK}{\gamma}\sum_n J_{n-1}(n\chi)\sin(n\xi) \tag{17}$$

Using Eq. (15) we get change in phase as

$$\frac{d\xi}{dz}=\frac{\Omega}{c}-\frac{k}{2\gamma^2}\left[1+K^2\right] \tag{18}$$