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Nonlinear dynamics and synchronization of saline oscillator's model

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ABSTRACT

The Okamura model equation of saline oscillator is refined into a non-autonomous ordinary differential equation whose coefficients are related to physical parameters of the system. The dependence of the oscillatory period and amplitude on remarkable physical parameters are computed and compared to experimental results in order to test the model. We also model globally coupled saline oscillators and bring out the dependence of coupling coefficients on physical parameters of the system. We then study the synchronization behaviors of coupled saline oscillators by the mean of numerical simulations carried out on the model equations. These simulations agree with previously reported experimental results.

1. Introduction

Nonlinear systems are common in nature. The dynamics of a broad variety of such physical, electromechanical, chemical and biological systems has been extensively studied in the literature and modeled by nonlinear ordinary differential equations [1–3]. Several models of noise have sometimes been added in such mathematical models to account for the random fluctuations induced by the environment [4–7]. Noisy synchronization of self-sustained oscillators has been studied so far in the literature. If the oscillators are coupled weakly enough and if they are also subject to random disturbances, or "noise", then the state of synchronization will not be constant or fixed. Instead, the oscillators may for a time become synchronized, but, due to the disturbances, may drop out of synchrony and then regain it, and so on.

Some nonlinear systems, called self-oscillatory systems [8–13], spontaneously exhibit regular rhythms. An example is the so-called saline oscillator. In the last three decades, the dynamics of the periodic fluid flow which occurs when a syringe (without plunger) at the center of its base, filled with salt solution and placed in a larger container filled with pure water has been considered by several authors like Okamura and Yoshikawa [14], Aoki [15], Miyakawa and Yamada [16,17]. This set-up known as saline oscillator has been shown to exhibit many features of nonlinear oscillators as limit-cycle, bifurcation of the oscillatory mode [18–22]. Okamura and Yoshikawa clarified the mechanism of rhythmic fluid flow in saline oscillator by performing the numerical computation of a three-dimensional Navier–Stokes equation. They concluded that pressure causes oscillation meanwhile viscosity and inertia depress oscillation. They also show that the saline oscillator can be modeled by the Rayleigh equation. Aoki proposed the so-called mathematical model of the density oscillator and analyzed the existence of a limit-cycle and a bifurcation to the oscillatory flow theoretically. Miyakawa and Yamada experimentally studied two coupled salt-water oscillators and observed synchronization phenomena. They also investigated the effect of viscosity on coupling behaviors and reproduced experimental results by numerical simulations using coupled nonlinear differential equations of the Rayleigh type. Málaga et al. [23] modeled the upward flow and downward one separately by two differential equations

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of the same form. Kano and Kinoshita [24] derived a sole model of a density oscillator by considering in addition to the up-flow and down-flow, the flow-reversal process. Most of works of the preceding researchers was therefore devoted to modeling of that curious hydrodynamic oscillator.

Yoshikawa et al. [25–27] found through semi-quantitative modeling that the essential features of saline oscillator are reproducible by the Rayleigh equation. Later, Okamura and Yoshikawa [14] model, through a full Navier–Stokes equation simulation, the saline oscillator with an ordinary differential equation of the same type. The present work is devoted, as one of the issues, on refining the model presented by Okamura by taking into account the implicit dependence on time of the density and the specific viscosity of the fluid within the orifice. In fact, if we make the wrong assumption that the density of the fluid passing through the orifice is almost constant at any time irrespective of the flow direction, then the oscillator can be modeled by a single differential equation, namely the Rayleigh equation; which seems to reproduce the experimentally observed oscillatory trend. That is why several investigators used a single Rayleigh equation to feature the saline oscillator [14,16,18]. It has been experimentally observed that when the specific viscosity of the salty solution is larger than that of the pure water, the duration of the downward flow is greater than that of the ascendant flow [17]. Such a feature, which cannot be accounted for by a single differential equation, has its essence in the alternate change of the fluid nature within the orifice. That is to say, the dependence of the density and the specific viscosity of the fluid in the orifice on time. We also derived the model equations of coupled saline oscillators with the same assumptions. The other issue of this work is the comparison of responses of the model equations with known experimental and theoretical results.

The rest of the paper is organized as follows: In Section 2, a description and derived equations of the density oscillator are presented. In Section 3, the dynamics of the density oscillator model is investigated through numerical simulations and the behaviors of the amplitude and the intrinsic period as function of model's physical parameters are reported. Section 4 is devoted to the derivation of model equations of coupled saline oscillators. Synchronization properties are then analyzed and compared to known results. Finally, a brief conclusion is given in Section 5.

2. Description of the system and governing equations

The investigated system called saline (or density) oscillator was first discovered and described in 1970 by the geophysicist Seelye Martin [28] who observed a rhythmic oscillation made of ascendant and descendant fluid flows through an experimental device consisting of a syringe without plunger, filled with saline solution, held in the vertical position, and then partially immersed in an outer vessel containing pure water. The rhythmic flow was also observed by Alfredsson and Lagerstedt [29] when a cup with a pinhole in its bottom was used instead of a syringe. In the present investigation, the saline oscillator is consisting of a large cylinder within which is held a cup with a pinhole in its base whose radii are respectively *c, b* and *a* as presented schematically in Fig. 1. The depth or thickness of the orifice is denoted by

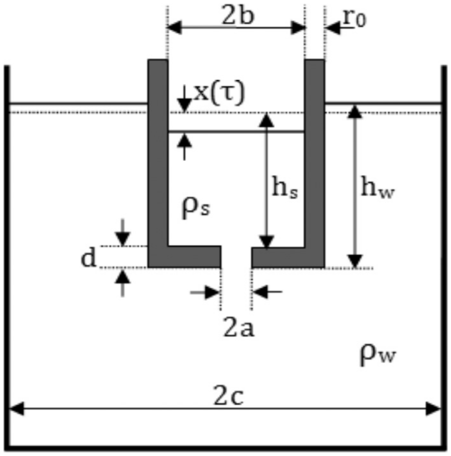

Fig. 1. Schema of experimental set-up for saline oscillator. The inner container is filled with salt water of density ρ_s and the large reservoir contains pure water of density ρ_w. The constant $h_s$is the initial height of salt water while h_w is the current height of pure water. Initially the level of pure water and that of the salt water are the same. $x(\tau)$ denote the shifting, at the time τ, of the current level of salt water from its initial level. Note that *a, b* and *c* are respectively radii of the orifice, the cup and the outer container. *d* is the depth of the orifice.

d. The outer container and the inner cup are filled with pure water and salt water with densities ρ_w and ρ_s respectively.

The flow within the orifice can be described by the Navier–Stokes equation and an equation of continuity. We suppose that the flow within the pinhole is parallel to the z axis and we neglect the dependence of the *z* component of the velocity on the *z* coordinate. The Navier–Stokes equation is then written in the cylindrical coordinate system whose origin is in the center of the bottom of the orifice. Following the formulation of Yoshikawa et al. [26], we evaluate the average of the Navier–Stokes equation over the volume of the tube. We take the pressure loss or the head loss [26] of the form suggested by Okamura and Yoshikawa [14], that is: $\Delta P^{*}(\tau) = \rho(\tau)(\beta\langle w\rangle - \gamma\langle w\rangle^3)d$, where $\langle w\rangle$ is the average of the *z* component of the velocity, β and γ are coefficients for the adjustment of the dimension which depend on the shape of the orifice and where ρ, which is the density of the fluid within the orifice, depends on the time τ. The pressure difference on both ends of the orifice is given by (see Fig. 1):

$$P(d,\tau) - P(0,\tau) = \rho_s g(h_s + x(\tau)) - \rho_w g h_w - \rho(\tau)(\beta\langle w\rangle - \gamma\langle w\rangle^3)d, \quad (1)$$

where the height h_w depends on $x(\tau)$; that is:

$$h_w = h_s + d - \frac{S_{in}}{S_{out}}x(\tau). \quad (2)$$

S_{in} and S_{out} respectively represent the free surfaces areas of salt water and pure one. Therefore, the equation describing the movement of the free surface of salty water (see Fig. 1), reduced from the Navier–Stokes equation, is given by

$$\ddot{x} = \left(\beta - \frac{8\nu}{a^2}\right)\dot{x} - \gamma\frac{b^4}{a^4}\dot{x}^3 - \frac{a^2}{b^2}\left(\frac{\rho_s}{\rho} + \frac{\rho_w}{\rho}\frac{S_{in}}{S_{out}}\right)\frac{g}{d}x - \frac{a^2}{b^2}\left(\frac{(\rho_s-\rho_w)h_s}{\rho d} - \frac{\rho_w}{\rho} + 1\right)g, \quad (3)$$