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Memory-induced sign reversals of the spatial cross-correlation for particles in viscoelastic shear flows

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ABSTRACT

The behavior of shear-induced cross-correlation functions between particle fluctuations along orthogonal directions in the shear plane for harmonically trapped Brownian particles in a viscoelastic shear flow is studied. A generalized Langevin equation with a power-law-type memory kernel is used to model the complex structure of the viscoelastic media. Interaction with fluctuations of environmental parameters is modeled by a multiplicative white Gaussian noise, by an internal fractional Gaussian noise, and by an additive external white noise. It is shown that the presence of a memory has a profound effect on the behavior of the cross-correlation functions. Particularly, memory-induced reentrant sign reversals of the spatial cross-moment between orthogonal random displacements of a particle are established, i.e., an increase of the memory exponent can cause the sign reversal from positive to negative, but by a further increase of the memory exponent a reentrant transition from negative to positive values appears. Similarities and differences between the behavior of the models with additive internal and external noises are considered. It is shown that additive external and internal noises cause qualitatively different dependencies of the cross-correlation functions on the time lag. The occurrence of energetic instability due to the influence of multiplicative noise is also discussed.

1. Introduction

Combined inertial and diffusive motion of Brownian particles in a flowing fluid (e.g. aerosols or dusty plasmas) has long been of interest for its important industrial and engineering applications [1]. Of particular interest are small mesoscopic systems such as colloidal particles, nanoparticles in solutions, or biological systems in cells, all of which are dominated by fluctuations [2] and can be considered as complex systems at some microscopic level. It is well known that a stochastic force (noise) can modify the behavior of nonequilibrium complex systems in a counterintuitive way and thus induce unexpected ordered outcomes such as stochastic resonance [3–6], ratchet effect [7–10], noise-enhanced stability [1,11,12], anomalous transient behavior of ecosystems [13], anomalous diffusion [14,15], etc. Particularly, experiments from many different areas reveal that anomalous diffusion with a mean-square displacement of particles $\langle r^2(t)\rangle \sim t^\alpha$, ($\alpha \neq 1$) is ubiquitous in nature, signaling that slow transport, ($\alpha < 1$), may be generic for complex heterogeneous materials [14]. Although the behavior of Brownian motion in quiescent fluids has been investigated in detail, the understanding of noise driven particle dynamics in flows is far from complete, in spite of its fundamental relevance and importance in microfluidic applications [2,16]. Recently, several studies have focused on the stochastic dynamics of underdamped Brownian particles trapped by harmonic potentials and exposed to shear flows [2,16–20]. The interest in harmonically trapped particles has been stimulated by some

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relatively new experimental techniques to trap mesoscopic particles, such as laser-optical tweezers [21,22], which allow direct observation and manipulation of individual particles under the influence of external forces of flows [16]. However, in papers [2,16–20] it is assumed that the interaction of Brownian particles with shear flow is characterized by Stokes friction. The latter is irrelevant for shear flow in viscoelastic media, where anomalous diffusion occurs [23,24]. Examples of such systems are supercooled liquids, colloidal suspensions, polymer solutions [25,26], the cytoplasm of living cells [27,28], and dusty plasmas [29]. Moreover, the previous calculations are based on models without multiplicative noise. It is important to note that multiplicative noise arises in a natural way in quantitative measurements with laser-optical tweezers, where the stiffness of the effective trapping potential may fluctuate [21]. Furthermore, multiplicative noise emerges in the description of complex systems as result of the elimination of fast degrees of freedom [30]. Therefore formalisms which are supposed to describe complex processes and which do that in terms of additive noise alone, may miss essential features of the problem. Recently, to overcome part of these problems the authors of Ref. [31] have considered a generalized Langevin equation (GLE) with a power-law-type memory kernel, which models the dynamics of an underdamped Brownian particle in a fluctuating harmonic potential well subjected to an oscillatory viscoelastic shear flow. The influence of a fluctuating environment is modeled by a multiplicative white noise (fluctuations of the stiffness of the trapping potential) and by an internal fractional Gaussian noise. It was shown that an interplay of shear flow, memory, and multiplicative noise can generate a variety of cooperation effects, such as energetic instability, multiresonance versus the shear frequency, memory-induced anomalous diffusion in the direction of the shear flow, etc. However, the paper [31] leaves open an important question, both from the theoretical and practical viewpoints, namely, what happens if the internal noise is replaced with an external noise.

Motivated by the theoretical results of Refs. [31] and [16–20,32], the present paper considers a model similar to the one presented in [31], except that the oscillatory shear flow is replaced with a time-independent shear flow as in [16–18,20] and that the additive internal noise is replaced with a superposition of an internal noise and a white external noise. The last mentioned modification enables to discern effects caused by internal and external noises, which may be an actual necessity in experiments. In the framework of this modified model, we study in more detail than in [31] the behavior of shear-induced cross-correlation functions between particle fluctuations along orthogonal directions in the shearplane. It should be noted that shear-induced cross-correlation functions of particle fluctuations are the main object of the experimental investigations reported in [20].

The main contribution of this paper is as follows. In the long-time limit, $t \to \infty$, we provide exact formulas for the analytical treatment of the dependence of particle cross-correlation functions on system parameters, such as the memory exponent, the shear rate, the damping coefficient, and the intensities of the multiplicative and additive noises. As our main result, memory-induced sign reversals of the spatial cross-moments between orthogonal random displacements of a particle are established. Particularly, it is shown that in the case of internal noise (without additive external noise) this highly unexpected effect occurs only if the intensity of fluctuations of the trapping potential is lower than a certain critical value. Let us note that in the case of Stokes friction the corresponding cross-moment is always positive. Moreover, it is shown that additive external and internal noises cause qualitatively different dependencies of the cross-correlation functions on the lag time.

The structure of the paper is as follows. In Section 2, on the basis of Ref. [31], we present the model investigated. Exact formulas for the analysis of the behavior of the cross-correlation functions are presented in Section 3. In Section 4 we analyze the behavior of the cross-correlation functions and expose the main results of this paper. Section 5 contains some brief concluding remarks.

Finally, to avoid misunderstandings, let us mention that the effect of memory-induced sign reversals of the spatial cross-moment is a novel effect not considered previously in Ref. [31].

2. Model

As in our previous work [31], we consider a Brownian particle of the unit mass ($m = 1$) suspended at the position $\mathbf{r} = (X, Y, Z)$ in a viscoelastic flow field with parallel streamlines in the x direction

$$\mathbf{v}(\mathbf{r}, t) = \rho Y(t)\mathbf{e}_x, \tag{1}$$

where $\mathbf{e}_x$ denotes the unit vector in the x direction and ρ is the shear rate. The particle is trapped by a harmonic potential with its minimum at $\mathbf{r}_0 = 0$,

$$U(\mathbf{r}) = \frac{\omega^2}{2}\mathbf{r}^2, \tag{2}$$

where ω is the trap frequency. As a model for such a system with memory, strongly coupled with a noisy environment, we consider a GLE with a fluctuating harmonic confinement potential $\tilde{U}(\mathbf{r})$

$$\ddot{\mathbf{r}}(t) + \gamma \int_0^t \eta(t-t')[\dot{\mathbf{r}}(t') - \mathbf{v}(\mathbf{r}(t'), t')]dt' + \nabla\tilde{U}(\mathbf{r}(t)) = \boldsymbol{\xi}(t), \tag{3}$$

where $\dot{\mathbf{r}}(t) \equiv d\mathbf{r}/dt$ and γ is the damping coefficient (friction coefficient) [31]. The fluctuating confinement potential $\tilde{U}(\mathbf{r})$ is assumed to be in the form

$$\tilde{U}(\mathbf{r}) = U(\mathbf{r}) + \frac{1}{2}\left(X^2 \cdot \xi_1^{(3)}(t) + Y^2 \cdot \xi_2^{(3)}(t) + Z^2 \cdot \xi_3^{(3)}(t)\right), \tag{4}$$

where $\boldsymbol{\xi}^{(3)}(t) = [\xi_1^{(3)}(t), \xi_2^{(3)}(t), \xi_3^{(3)}(t)]$ is a white Gaussian noise with the properties

$$\langle \boldsymbol{\xi}^{(3)}(t)\rangle = 0, \ \left\langle \xi_i^{(3)}(t)\, \xi_j^{(3)}(t')\right\rangle = 2D\delta_{ij}\delta(|t-t'|), \tag{5}$$

with δ_{ij} denoting the Kronecker symbol.

The noise $\boldsymbol{\xi}^{(3)}(t)$ with intensity D is assumed as statistically independent from the noise $\boldsymbol{\xi}(t) = [\xi_1(t), \xi_2(t), \xi_3(t)]$. Depending on the physical situation, the zero-centered driving noise $\boldsymbol{\xi}(t)$ can be regarded either as an internal noise, in