Chaos, Solitons and Fractals 81 (2015) 463–472

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Trajectory statistics and turbulence evolution

M. Vlad*, F. Spineanu

National Institute of Laser, Plasma and Radiation Physics, Atomistilor 409, 077125 Magurele, Bucharest, Romania

ARTICLE INFO

Article history:
Available online 14 September 2015

Keywords:
Test particle statistics
Turbulence
Self-organization
Vorticity
Euler fluid

ABSTRACT

The aim of this paper is to understand the tendency to organization of the turbulence in two-dimensional ideal fluids. A different perspective on vorticity separation and on the inverse cascade of energy yields from this study. Trajectory trapping or eddying appears to be strongly connected to these nonlinear processes. The statistics of the trajectories of the vorticity elements in a turbulent state is studied using a semi-analytic method. We show that the separation of the positive and negative vorticities is due to the attraction produced by a large scale vortex on the small scale vortices of the same sign. More precisely, a large scale velocity is shown to determine average transverse drifts, which have opposite orientations for positive and negative vorticity. They appear only in the presence of trapping and lead to energy flow to large scales due to the increase of the circulation of the large vortex. Recent results on drift turbulence evolution in magnetically confined plasmas are discussed in order to underline the idea that there is a link between the inverse cascade and trajectory trapping. The physical mechanisms are different in fluids and plasmas due to the different types of nonlinearities of the two systems, but trajectory trapping has the main role in both cases.

1. Introduction

The two-dimensional fluid turbulence has represented during more than 70 years an active field of research (see the review papers [1–4] and the references therein). It has many applications in different areas as fluid dynamics, meteorology, oceanography, fusion plasmas, superfluids, superconductors and astrophysics, in spite of the fact that it provides only idealized models for physical systems that are always three-dimensional.

The two-dimensional turbulence has a self-organizing character, which is related to the invariance of both energy and enstrophy in ideal (inviscid) fluids. Numerical studies of the decaying turbulence clearly show this property and the associated scaling behavior [5–7]. The enstrophy has a direct cascade (to small scales), but with a complex evolution characterized by the presence of inverse cascade in isolated regions [8,9]. The energy has an inverse cascade (to large scales) that leads to the emergence of large quasicoherent vortices. The process of self-organization can continue until the coherent vortices reach the size of the system [10,11]. This behavior was explained in the representation of point-like vortices by a negative temperature [12,1,13] or by the property of self-duality of the associated field theoretical model [14]. The latter approach was extended to models of planetary atmosphere and of magnetized plasmas [15].

This paper deals with the turbulent states and studies the self-organization during its initial stage, before the emergence of large coherent vortices of the system size. Our approach belongs to the Lagrangian statistical formalism, which is based on determining the statistics of test particle (tracer) trajectories (see the review paper [4]).

We show that trajectory trapping or eddying in the structure of the turbulence is the main physical reason for the strong nonlinear effects that are observed in two-dimensional ideal fluids. This conclusion is drawn from a

---

* Corresponding author. Tel.: +40721844112; fax: +40214574493.
E-mail address: madalina.vlad@inflpr.ro, madalina.olimpia.vlad@gmail.com (M. Vlad).

http://dx.doi.org/10.1016/j.chaos.2015.08.023

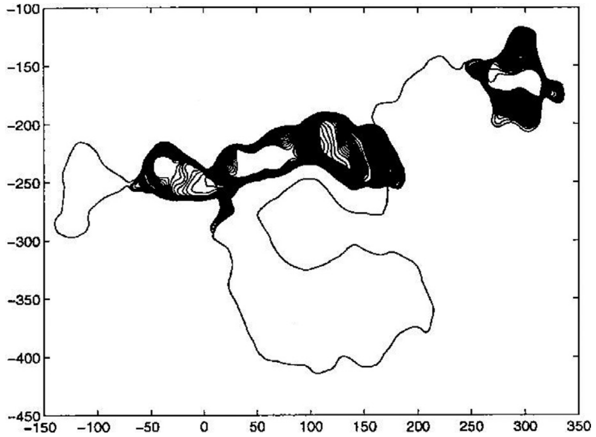

Fig. 1. A typical trajectory obtained from Eq.(1) for$K = 10$, $V_d = 0$ and the stream function used in the simulations presented in [30].

study of the statistics of test particles (tracers) in turbulent Euler fluids.

This study is based on a series of recent results on the statistical properties of test particle trajectories in incompressible two-dimensional velocity fields. Numerical simulations have shown that trajectories are complex, as they have both random and quasi-coherent aspects. A typical trajectory, shown in Fig. 1, is a random sequence of long jumps and trapping events that consists of winding on almost closed paths. Analytical methods that describe the statistics of these trajectories were developed [16,17] and used for understanding various aspects of turbulent transport. It was shown that they provide a very good description of the nonlinear effects produced by trajectory trapping or eddying and reasonably accurate quantitative results for the diffusion coefficients and for other statistical averages.

The conclusion of these studies is that trajectory trapping or eddying leads to nonstandard statistics: memory effects (represented by long time Lagrangian correlation), strongly modified transport coefficients and non-Gaussian distributions of displacements. It was also shown that trapping determines a large degree of coherence in the sense that bundles of trajectories that start from neighboring points remain close for very long time compared to the eddying time. Trapped trajectories form quasi-coherent structures similar to fluid vortices. Extensive theoretical [18–22] and numerical studies [23–25] have contributed significantly in the last decades to the understanding of the turbulent transport in laboratory or space plasmas, in fluids or in stochastic magnetic fields.

A strong connection between test particle trapping and turbulence evolution was found in [26] from a study of test modes on turbulent plasmas. Analytical results that are in agreement with numerical simulations were obtained, and they allowed to deduce a new physical perspective on the nonlinear process of generation of large scale correlations (inverse cascade) and of zonal flow modes. Essentially, they are effects of ion trajectory trapping or eddying.

We show here that trapping has an essential role in two-dimensional fluid turbulence. A nonlinear effect produced by trapping of the vorticity elements brings a different perspective on the separation of positive and negative vorticity and on the inverse cascade of the energy.

The paper is organized as follows. The problem of test particle or tracer transport is defined in Section 2.1. Section 2.2 contains a short presentation of the analytical statistical approach, the decorrelation trajectory method (DTM). The effects of trapping or eddying on tracer transport and on the statistical characteristics of the trajectories are discussed in Section 3. This section contains a review of the previous work and new results on the modifications of the trajectory structures determined by an average velocity. The effects of trapping on the decaying two-dimensional turbulence in ideal fluids are analyzed in Section 4.1. The physical process that determines the separation of the positive and negative vorticities and leads to the inverse cascade of the energy is identified in this approach. The average speed of vorticity separation is estimated. Section 4.2 is a short discussion on recent results on plasma turbulence evolution, which shows that trajectory trapping has an essential role in the inverse cascade, although the physical mechanism is completely different. The conclusions are summarized in Section 5.

2. Test particle transport and the statistical method

2.1. The problem

The problem of test particle or tracer advection in two-dimensional incompressible velocity fields is described by the stochastic equation:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}[\mathbf{x}(t), t] + V_d \mathbf{e}_y, \tag{1}$$

where $\mathbf{x}(t)$ is the trajectory in the plane $(\mathbf{e}_x, \mathbf{e}_y)$, $\mathbf{v}(\mathbf{x}, t)$ is the stochastic velocity and $V_d \mathbf{e}_y$ is an average velocity that is taken in the $\mathbf{e}_y$ direction. The velocity is a continuous function of $\mathbf{x}$ and t in each realization and it determines an unique trajectory as the solution Eq. (1) with the initial condition $\mathbf{x}(0) = \mathbf{0}$. The stochastic velocity $\mathbf{v}(\mathbf{x}, t)$ is incompressible $[\nabla \cdot \mathbf{v}(\mathbf{x}, t) = \mathbf{0}]$ and is represented by a scalar field, the stochastic potential or stream function

$$\mathbf{v}(\mathbf{x}, t) = \mathbf{e}_z \times \nabla\phi(\mathbf{x}, t) = (-\partial_y \phi(\mathbf{x}, t),\ \partial_x \phi(\mathbf{x}, t)). \tag{2}$$

The potential $\phi(\mathbf{x}, t)$ is considered to be a stationary and homogeneous Gaussian stochastic field, with zero average and given two-point Eulerian correlation function (EC)

$$E(\mathbf{x}, t) \equiv \langle \phi(\mathbf{x}', t')\, \phi(\mathbf{x}' + \mathbf{x}, t' + t) \rangle \tag{3}$$

where $\langle \cdots \rangle$ denotes the statistical average over the realizations of $\phi(\mathbf{x}, t)$ or the integral over $\mathbf{x}'$ and t'. The main parameters of the EC are: the amplitude of the potential fluctuations $\beta^2 = E(\mathbf{0}, 0)$, the correlation length λ_c and the correlation time τ_c, which are the characteristic length and time of the decay of the function $E(\mathbf{x}, t)$. The EC's of the velocity components are obtained as space derivatives of $E(\mathbf{x}, t)$ and the amplitude of the stochastic velocity is $V = \beta/\lambda_c$.

Starting from the above statistical description of the stochastic potential and from an explicit EC one has to determine the statistical properties of the trajectories. This problem is nonlinear due to the space dependence of the potential, which leads to $\mathbf{x}$ dependence of the EC (3).

The equation of motion is nonlinear due to the space dependence of the velocity field. The trajectories are solutions