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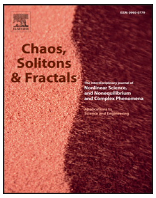

Stability of freely cooling granular mixtures at moderate densities

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ABSTRACT

The formation of velocity vortices and density clusters is an intriguing phenomenon of freely cooling granular flows. In this work, the critical length scale L_c for the onset of instability is determined via stability analysis of the linearized Navier–Stokes hydrodynamic equations of d-dimensional granular binary mixtures at moderate densities. In contrast to previous attempts, the analysis is not restricted to nearly elastic systems since it takes into account the nonlinear dependence of the transport coefficients and the cooling rate on the collisional dissipation. As expected from previous results obtained in the very dilute regime, linear stability shows $d-1$ transversal (shear) modes and a longitudinal ("heat") mode to be unstable with respect to long enough wavelength excitations. The theoretical predictions also show that the origin of the instability is driven by the transversal component of the velocity field that becomes *unstable* when the system length $L > L_c$. An explicit expression of L_c is obtained in terms of the masses and diameters of the mixture, the composition, the volume fraction and the coefficients of restitution. Previous results derived in the limit of both mechanically equivalent particles and low-density mixtures are consistently recovered. Finally, a comparison with previous theoretical works which neglect the influence of dissipation on the transport coefficients shows quantitative discrepancies for strong dissipation.

1. Introduction

Granular media are involved in many industrial and natural phenomena. In fact, it has been estimated that granular media is the second most used type of material in industry after water [1]. This is perhaps the main reason for which the study of granular matter has attracted the attention of physicists and engineers in the past few years. Although granular media form an extremely vast family constituted by grains of different sizes and shapes, all these systems share relevant features. In particular, when granular materials are externally excited (rapid flow conditions), they behave like a fluid. In this regime, binary collisions prevail and hence, kinetic theory may be considered as a quite useful tool to describe the kinetics and hydrodynamics of the system. The main difference with respect to ordinary or molecular fluids is that granular systems are constituted by macroscopic grains that collide inelastically so that the total energy decreases with time. In this context, a granular fluid can be considered as a *complex* system that inherently is in a non-equilibrium state. In the case that the system is heated by an external driving force that compensates for the energy dissipated by collisions, a non-equilibrium *steady* state is achieved. Under these conditions, some attempts have been recently made to formulate a *fluctuation–response* theorem based on the introduction of an effective temperature [2–9]. The generalization of the equilibrium fluctuation–response theorem to non-equilibrium states has been also confirmed in real experiments of intruders in driven granular fluids [10]. Another interesting experiments in granular matter have studied the

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response of a sheared granular medium in a Couette geometry [11] and the behaviour of a freely rotating asymmetric probe immersed in a vibrated granular media [12].

On the other hand, although significant progresses have been made in the past on the understanding of granular flows, there are still important open challenges in the research of granular gases. One of the main reasons for which the theoretical description of these systems is quite intricate is that the number of relevant parameters needed to describe them is relatively large. This gives rise to a wide array of complexities that arise during the derivation of kinetic theory models. Thus, in order to gain some insight into the description of these systems under real conditions, one usually models a granular fluid as a system composed by *smooth* hard spheres or disks with inelastic collisions. In this simplest model, the inelasticity of collisions is only accounted for by a (positive) *constant* coefficient of normal restitution $\alpha \leq 1$ that only affects the translational degrees of freedom of grains. Nevertheless, in spite of the simplicity of the model, it has been shown as a reliable prototype to explain some of the physical mechanisms involved in granular flows, especially those directly related to collisional dissipation.

One of the most characteristic features of granular fluids is the spontaneous formation of velocity vortices and density clusters in freely cooling flows (homogeneous cooling state, HCS). The origin of this kind of instability is associated with the dissipative nature of collisions and is likely the most characteristic feature that makes granular flows so distinct from ordinary (elastic) fluids. Detected first by Goldhirsch and Zanetti [13] and McNamara [14] in computer simulations, the instabilities in a free granular fluid can be well described by a linear stability analysis of the Navier–Stokes hydrodynamic equations. This analysis provides a critical length L_c so that the system becomes unstable when its linear size is larger than L_c. In the case of a monodisperse low-density granular gas, the dependence of L_c on the coefficient of restitution obtained from the (inelastic) Boltzmann kinetic equation [15,16] compares quite well with numerical results [17] obtained by using the direct simulation Monte Carlo (DSMC) method [18]. For higher densities, theoretical results for L_c based on the (inelastic) Enskog equation [19] shows an excellent agreement with molecular dynamics (MD) simulations for a granular fluid at moderate density [20,21]. The stability analysis reported in Ref. [19] extends to finite dissipation some previous attempts [22,23] carried out in the context of the Enskog kinetic theory but neglecting any dependence of the pressure and the transport coefficients on inelasticity. Nevertheless, while the study of the stability of the HCS has been widely covered in the case of granular fluids, much less has been made in the important subject of granular mixtures (namely, systems composed by grains of different masses, diameters, composition).

Needless to say, the analysis of the stability of the HCS for polydiperse granular systems is much more complicated than for a single granular gas. Not only the number of transport coefficients involved in the determination of the critical size L_c is higher than for a monodisperse gas but also they depend on more parameters, such as the set of coefficients of restitution characterizing the binary collisions between different species. Many of the early attempts [24–27] to obtain the Navier–Stokes coefficients of granular mixtures were performed by assuming the equipartition of granular energy. However, given that the lack of energy equipartition [28] has been widely confirmed by computer simulations [29–33] and observed in real experiments of agitated mixtures [34,35], the hypothesis of energy equipartition can only be acceptable for nearly elastic systems. In fact, in those previous works [24–27] the forms of the transport coefficients are the same as those obtained for ordinary mixtures [36] and the influence of inelasticity is only considered in the presence of a sink term in the energy balance equation. A more rigorous derivation of linear transport for granular mixtures has been made by Garzó and Dufty [37] in the dilute regime and more recently by Garzó, Dufty and Hrenya [38–40] for moderate densities. In these works, given that non-equipartition effects on transport have been considered, the corresponding Navier–Stokes transport coefficients exhibit an intricate nonlinear dependence on the coefficients of restitution of the mixture. As for single granular gases, the theoretical results (which have been obtained in the so-called fist Sonine approximation) compare in general quite well with computer simulations [41–46] for conditions of practical interest, such as strong inelasticity.

The knowledge of the Navier–Stokes transport coefficients of granular mixtures opens the possibility of obtaining the critical length L_c from the (linear) stability analysis of the hydrodynamic equations. For dilute systems, the theoretical predictions of kinetic theory [47,48] for L_c has been shown to agree very well with the DSMC simulations of the Boltzmann equation [48]. On the other hand, in spite of the explicit knowledge of the Enskog transport coefficients for a granular mixture [38–40], I am not aware of any previous solution of the linearized hydrodynamic equations for moderately dense granular mixtures. The goal of this paper is to perform a linear stability analysis around the HCS in order to identify the conditions for stability as functions of the wave vector, the volume fraction, the dimensionality of the system d and the parameters of the mixture (masses, sizes, composition and the coefficients of restitution). As expected, the stability analysis shows $d-1$ transversal (shear) modes and a longitudinal heat mode to be unstable with respect to long wavelength excitations. In addition, the results also show that the origin of the instability lies in the transversal shear mode (except for quite large dissipation) and hence, for sizes of the system larger than the critical length L_c the transversal velocity becomes unstable. As for dilute mixtures [48], theoretical predictions for L_c compare well with recent MD simulations of hard spheres [49]. A preliminary short report of some of the results presented here has been given in Ref. [49].

The plan of the paper is as follows. First, in Section 2 the hydrodynamic equations and associated fluxes to Navier–Stokes order are recalled. The explicit dependence of some of the transport coefficients on dissipation is illustrated for different systems showing that the influence of inelasticity on transport is in general quite significant. Section 3 is devoted to the linear stability analysis around the HCS. This Section presents the main results of the paper. The dependence of the critical size L_c on the parameter space is widely investigated in Section 4 by varying the parameters of the system in the case of a common coefficient of restitution ($\alpha_{11} = \alpha_{22} = \alpha_{12} \equiv \alpha$). The paper is closed in Section 5 with a brief discussion of the results.