Chaos, Solitons and Fractals 78 (2015) 39–46

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Noise dependency of algorithms for calculating fractal dimensions in digital images

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ARTICLE INFO

Article history:
Received 24 April 2015
Accepted 5 July 2015
Available online 25 July 2015

MSC:
00-01
99-00

Keywords:
Fractals
Fractal dimension
Image analysis
Noise
Tug of war

ABSTRACT

Fractal properties of real world objects are commonly examined in digital images. Digital images are discrete representations of objects or scenes and are unavoidably contaminated with noise disturbing the representation of the captured objects. We evaluate the noise dependency of frequently applied algorithms for the calculation of the fractal dimension in digital images. Three mathematically defined fractals (Koch Curve, Sierpinski Gasket, Menger Carpet), representative for low, middle and high values of the fractal dimension, together with an experimentally obtained fractal structure were contaminated with well-defined levels of artificial noise. The Box-Counting Dimension, the Correlation Dimension and the rather unknown Tug-of-War Dimension were calculated for the data sets in order to estimate the fractal dimensionality under the presence of accumulated noise. We found that noise has a significant influence on the computed fractal dimensions (relative increases up to 20%) and that the influence is sensitive to the applied algorithm and the space filling characteristics of the investigated fractal structures. The similarities of the effect of noise on experimental and artificial fractals confirm the reliability of the obtained results.

1. Introduction

The concept of fractal geometry, introduced by Benoit B. Mandelbrot in 1983, has become very useful for estimating the complexity of natural objects. Fractal geometry becomes apparent in natural and artificial structures in a wide range of environments and forms. The fractal dimension of an object is an important measure of the intrinsic dimensionality. The fractal dimension coincides with the irregularity and the roughness of a self-similar object, and indicates how much space is filled by the object.

The interpretation of real objects as fractals has proved to be of great interest in texture analysis, segmentation and the analysis of digital images in a broad field of different applications [1]. The determination of fractal dimensions of medical images, especially for cancer, have been carried out very successfully, e.g. the investigation of leukemic cells [2], dysplastic lesions of cervix uteri [3], gallbladder adenocarcinomas [4], retinoid-induced differentiation of cancer cells [5], nuclear pleomorphism [6], pigmented skin lesions [7,8], malignant melanoma [9], mammographic density of breast carcinoma [10], MCF-7 breast cancer cells [11], breast epithelial cell nuclei [12], fractal growth patterns of breast cancer cells [13] or the invasiveness of mouse melanoma cells [14–18].

Although extensive research work has been carried out concerning calculations and interpretations of fractal dimensions, little attention has been paid to practical limitations of digital images. A digital image is a discrete, two dimensional representation of a three dimensional object with a single image pixel as the smallest entity. Digital images can be obtained from digital cameras, video cameras, microscopes, scanning devices or sophisticated devices using magnetic

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http://dx.doi.org/10.1016/j.chaos.2015.07.004
0960-0779/

spin resonance or fluorescence tracer methods. These devices mainly include optical and electronic techniques and therefore, several sources of noise can emerge. Examples are Poisson or Bose–Einstein distributed light sources of a microscope and Johnson- or Shotnoise of any electronic device. Additional errors can occur during image capturing, transmission, compression or image processing. Thus, it is mandatory to keep the signal to noise ratio as high as possible in order to ensure a high image quality. Noise may lead to a false or incomplete reproduction of the real object in a digital image. This means that pixels are not correctly associated with the real object or the background. Some background pixels may appear as object pixels or vice versa. In this sense, digital images from real objects can be interpreted as being measurements or estimations.

Furthermore, the representation of natural objects in a digital form is limited to a finite range of orders of magnitude. The size of a single pixel of an image is the lower limit and the size of the image itself is the upper limit. Hence, the calculation of the fractal dimension is restricted to a small range of magnitudes and the theoretical infinitesimal limit cannot be calculated because of the finite size of a single image pixel.

Since neither the Hausdorff measure nor the Hausdorff dimension are compatible with digital images, several more suitable methods, such as the very popular Box-Counting Dimension or the Correlation Dimension, are common practice in various research fields (e.g. health science, geography or geophysics). Standard approaches are typically based on power-law relationships between measured values and scaling variables. However, a crucial point is the actually implemented scanning (or raster-) technique which defines how the measured values are extracted from digital images. We carefully selected the Box-Counting Dimension, the Correlation Dimension and the rather unknown Tug-of-War approach because each method corresponds to a unique scanning technique.

In order to evaluate the influence of noise on the calculation of the fractal dimension in digital images our contribution in this study is two-fold. First, we prepared four image sequences comprising of three mathematically well defined fractals (with low, middle and high theoretical fractal dimensions) and one experimentally gained fractal structure with increasing levels of artificially added noise. This simulates the ubiquitous condition that digital images are always contaminated with more or less noise. Second, we calculated the fractal dimensions for each image sequence by means of different algorithms. Here, we focus on the Box-Counting Dimension, the Correlation Dimension and the Tug-of-War approach.

This paper is structured in the following way: Section 2 describes the preparation of the data sets, including the generation of theoretical fractals and the process of adding noise. In Section 3 we give an overview on the different approaches to calculate the fractal dimension. The results are revealed in Section 4 and their implications are discussed in Section 5.

2. Data preparation

In order to evaluate noise effects as inherent properties of digital images we prepared four image sequences, comprising of three artificial fractals (Menger Carpet, Sieprinski Gasket, Koch Curve) and one experimentally obtained digital image of a neuronal cell. Three mathematical fractals are representative for low, middle and high values of the fractal dimension with theoretical values of 1.26, 1.58 and 1.89, respectively. Artificial noise was added to the images in discrete steps in order to prepare four image-sequences, each containing 100 images with increasing levels of noise (see, Fig. 1).

2.1. Artificial fractals

The Menger Carpet, the Sierpinski Gasket and the Koch Curve were generated by a deterministic iterated function system [19] (IFS). The Menger Carpet is the two-dimensional analogue of the three-dimensional Menger Sponge and is also designated as Sierpinski Carpet [20]. Two-dimensional digital images of these fractals were iteratively constructed with a resolution of 1024×1024 or 1458×1458 pixels. The iterations were stopped as the fine details of the fractals converged toward the single pixel resolution. Further iterations would have caused artificial effects such as merging together distinct lines. The actual number of iterates was 8 for the Sierpinski Gasket, 5 for the Menger Carpet and 8 for the Koch Curve. The artificial noise-free fractals are shown in the first column in Fig. 1(a)–(c).

2.2. Digital image of a neuronal cell

It is well known that the morphology of neuronal cells show fractal properties [21] and therefore, we decided to include such an image, representative for natural fractal-like objects. A representative sample image of a neuronal astrocyte cell, showing the branched structure of dendrites was taken from the CCDB data base (http://ccdb.ucsd.edu/CCDBWebSite/index.html). This sample image was taken with a confocal microscope (Bio-Rad Microscience Ltd, Bio-Rad Radiance 2000, Hemel Hempstead) and had a resolution of 1024×1024 pixels. The original image in 8-bit grey level format was segmented by using the image processing software IQM [22] (see, e.g. http://sourceforge.net/projects/iqm).

2.3. Generation of noise

For each fractal a series of images with different levels of noise, represented by variance values σ^2 in the range [0, 100], was constructed. We did not simply add randomly distributed object pixels, e.g. by the application of suitable blurring operators because a random distributed addition of object pixels would alter the total number of image pixels proportionally to the pre-set variance. Hence, the computed values for fractal dimensions would be increased, simple due to the accumulation of object pixels. In order to circumvent this effect, the noise-contaminated images were generated by holding the total number of object pixels constant [15,18]. Individual object pixels were translocated to new locations by using a statistical approach. Statistical probabilities define the translocation distance d, with $d \geq 0$, and translocation angle $\varphi_i \in \{0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°\}$. A Gaussian and a negative exponential distribution were utilized for the calculation of distances d, and a uniform distribution was