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# Effect of local information within network layers on the evolution of cooperation in duplex public goods games

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ABSTRACT

Traditional works of public goods game (PGG) are often studied in simplex networks where agents play games through the same type of social interactions. In order to promote cooperation against the defection in PGGs in simplex network environment, many mechanisms have been proposed from different perspectives, such as the volunteering mechanisms, and the punishment and reward approaches. However, due to diverse types of interactions between agents in reality, the study of PGG should also consider the characteristic of multiplexity of networks. Hence, we firstly model the public goods game in the duplex network (for simplification of analysis, the duplex network is considered), in which agents have two types of social interactions, and thus the network is modeled as two network layers. This type of PGG is naturally named as duplex public goods game (D-PGG), in which agents can select one of the network layers to allocate their limited resources. Then for the new game environment (D-PGG), we propose a novel perspective to promote cooperation: degrading the information integrity, i.e., agents get information just from one network layer (local information) rather than from the whole duplex network (global information) in the evolution process. Finally, through theoretical analyses and simulations, we find that if agents imitate based on the local information of the payoff in the evolution, cooperation can be generally promoted; and the extent of promotion depends on both the network structure and the similarity of the network layers.


## 1. Introduction

Public goods game (PGG), which is a multi-person Prisoner's Dilemma, is often employed as a typical model for studying grouped cooperation in the society [1–6]. In a traditional PGG, agents choose whether to invest in a public pool, then the total contributions in the public pool are multiplied by an enhancement factor, and finally the obtained payoffs will be divided equally to all the participants irrespective of their contributions [7]. There is an equilibrium when all the agents refuse to invest [7], which, however, contradicts with the cooperative phenomena observed in reality [8]. There have been many studies trying to explain the maintenance of cooperation in the society [9–14]; and one of the most important methods is the evolutionary game theory [15]. Furthermore, the network of relationships for agents' interactions is also considered as a crucial reason for the evolution of cooperation [16,17]. Hence, studying evolutionary games in structured population becomes an important direction for the study of PGG.

Although many mechanisms have been proposed to promote the cooperation in PGG, most of them focus on the PGGs in *simplex networks* where the social interactions are of the same type [12–17]. However, there are diverse types of interactions between agents in reality [18–24]. Agents should make decisions on different types of interactions simultaneously, and thus the resources of agents should be put into one specific type of interactions while agents only have limited

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amount of resources [17]. Hence, we study the public goods game with different types of social interactions in this article; to simplify the model, we focus on the PGGs in *duplex networks* which are composed of two types of interactions and can be regarded as two network layers where each network layer is composed of interactions of the same type. For instance, the students in a university may have different interactions with their classmates and friends; hence, the relations of classmate and friend among the same set of students comprise two network layers of a duplex network, respectively. Then since they have limited resources, e.g., limited time and limited energy, each of them needs to make a selection between study and discussion with classmates and having fun with friends to put into the resources in a specific period. (Note that it is assumed that the limited resources cannot be split in each round of the game.)

The public goods game with limited resources in duplex networks is named as *duplex public goods game* (D-PGG). In a D-PGG, agents choose public pool in one network layer to allocate their limited resources rather than choosing to invest or not [17,25]; and then the total payoffs will be equally divided and then be returned to the agents and their neighbors in the corresponding network layer.

Generally, when agents play multiple rounds of games in social networks, they usually make decision by taking neighbors' decisions into account as they expect to improve their payoffs [16,17]. For example, in traditional PGGs, i.e., PGGs in simplex networks, the agents can imitate their neighbors' decisions based on the obtained information about their neighbors' payoffs [17,25]. Then in the introduced D-PGG, agents can have two types of referential information, local information and global information, according to the multiplexity of the network:

- *Local information*: the amount of payoffs agents obtained just from one of the network layers.
- *Global information*: the total amount of the payoffs agents obtained from all network layers.

Hence, the effect of different types of referential information on the evolution of cooperation is important for the study of D-PGG and thus has been investigated in this article.

Through theoretical analyses and simulations, we mainly obtain the following observations: (1) the local information for imitation can promote cooperation in D-PGG with different settings of network structures, games and evolution rules. (2) the extent of promotion is relevant to the structure of the two network layers and the similarity between them. For instance, in the duplex network with network layers $NL_1$ and $NL_2$, if network layer $NL_1$ is a regular network [26] and network layer $NL_2$ is a partial duplicate of random network [26] and $NL_1$, the more similar the two network layers are, the higher level of cooperation emerges. Then if network layer $NL_1$ is a scale-free network [27], the opposite result emerges: the more similar the two network layers are, the lower level of cooperation emerges. Moreover, we find if the duplex network consists of two network layers of random network structure, the similarity makes little difference.

We believe this study of D-PGG provide a novel way to understand the maintenance of cooperation: the cooperation can be promoted by degrading the information integrity. Furthermore, considering the social dilemma situation that agents need to coordinate with each other between the individual and collective interest, this study could also help the construction of multiplex societies in reality.

## 2. Related work

### *2.1. Promoting cooperation from the perspective of game rule in evolutionary games*

***Punishment:*** Punishment is often regarded as a common and effective mechanism for promoting cooperation in PGGs [28–35].

The previous studies on punishment can be classified into two types, which are the monetary punishment [30] and the non-monetary punishment [31]. Monetary punishment can make effect on promoting cooperation through the sanction on defectors' personal payoff, while the non-monetary punishment sanction is often conducted by social measures like social ostracism or exclusion. Helbing et al. [30] study the effect of monetary punishment on promoting cooperation in spatial public goods games. Two additional strategies, punishing cooperators (PC) and punishing defectors (PD) are incorporated in this work and are tested respectively. The payoff of defectors can be obtained by subtracting the punishment fines by punishing participants in the games. This work indicates that the cooperation can be promoted by increasing the fine with the PC strategy, while the cooperation shows a reentrant transition as the fine is increased with the PD strategy. Cinyabuguma et al. [31] propose a mechanism that allows the participants in the public goods games to perform majority voting to expel free-riders. This mechanism is a non-monetary punishment that using social measures. The authors find that the expelled member of the group is often the one who contributed the lowest or the second lowest, and even if it still remains in the next round it will increase its contribution to the public.

Punishment mechanisms can also be classified by the type of punishment actor: peer punishment, which is performed by the game participants [31,34], and pool punishment [32,33], which is conducted by a third party in the society. We mainly introduce the studies on pool punishment as follows. Traulsen et al. [32] design an economic experiment based on the public goods game, which indicates that pool punishment is preferred by humans when both punishment are concurrently available. Szolnoki et al. [33] study the efficiency of pool punishment in the spatial public goods game, in which three strategies, defector, cooperator, and pool punisher coexists. They find the pool punishment is less effective compared with peer punishment because of the invasion by cooperators into the territories of pool punishers, and then if the punishments are with high fines, the three strategies will coexist as a rock-paper-scissors type of cyclic.

Moreover, the punishment in optional public goods game has also been studied [34,35], in which the players have the option to participant in the game. Wang et al. [34] study the punishment in the optional public goods games, where four strategies, cooperator, defector, punisher and loner are considered. In their study, under the condition of bounded rationality, two equilibriums have been discovered; one of the equilibriums comprises punisher and cooperators, and the other one comprises the three types of participants without