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An approach to achieve modified projective synchronization between different types of fractional-order chaotic systems with time-varying delays

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ABSTRACT

In this paper, modified projective synchronization (MPS) for different fractional-order chaotic systems with variable time delays is investigated. By using active control method, and analysis of the error dynamical systems, a suitable controller for achieving the modified projective synchronization is given. Finally the proposed method illustrated by two examples in two conditions and performance of proposed method is shown.

1. Introduction

In recent years, study on the dynamics of fractional-order differential systems has greatly attracted interest of many researchers. In the last two decades, fractional calculus has been applied in an increasing number of fields. Compared with the classical integer order models, fractional-order derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes. With the introduction of fractional derivatives, it was proved that many fractional-order differential systems behave chaotically, such as fractional-order Chuas circuit [1], fractional-order Rössler system [2], fractional-order Chen system [3] and fractional-order Lü system [4] and fractional-order Lorenz system [5]. There are many investigations on the synchronization of two identical or different fractional-ordered chaotic systems without delays [6–10]. The control and synchronization of time-delay systems have received increasing attentions [11–14]. However, there are few results in the synchronization of a fractional-order chaotic systems with time delay [15] also there is not any specific study on synchronization of time-delay fractional-order chaotic systems with time-varying time delay.

Delay differential equation (DDE) is a differential equation in which the derivative of the function at any time depends on the solution at previous time. Introduction of delay in the model enriches its dynamics and allows a precise description of the real life phenomena. DDEs have proven useful in control systems [16], lasers, traffic models [17], metal cutting, epidemiology, neuroscience, population dynamics [18]. In DDE one has to provide history of the system over the delay interval $[-\tau, 0]$ as the initial condition. Due to this reason delay systems are infinite dimensional in nature. Baleanu and co-workers studied inclusion of delay in the fractional variational principles [19] and have given existence and uniqueness theorem for delay differential equation arising in this scheme [20,21]. Because of the infinite dimensionality the DDEs are difficult to analyze analytically [22] and the numerical solutions therefore, play an important role.

There are many types of synchronization in chaotic systems [23,24]. Recently, a more general form of synchronization scheme, called modified projective synchronization, has been investigated [25,26], where the responses of the

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synchronized dynamical states synchronize up to a constant scaling matrix. This kind of synchronization is more general kind of synchronization in chaotic systems, which contains other synchronization type, such as projective synchronization, complete synchronization and anti-synchronization. There are some studies in modified projective synchronization for fractional-order chaotic systems [27], but there are few studies on modified projective synchronization for fractional-order chaotic systems with time delays until now. In this paper, we consider the modified projective synchronization for fractional-order chaotic systems with time delays in drive and response system where this time delays can be as the same or different. Also we suppose that time delays are stochastic function of time which means that at any time fractional derivate of drive and response systems depends on its solution at previous time and this time is not fixed time and changes stochastically. In special cases of modified projective synchronization, we show the synchronization of two fractional-order chaotic systems.

This paper is organized as follows. We begin with the preliminary relations in Section 2, where the definitions of fractional-order operator, stability condition of fractional-order systems, numerical algorithm for simulation of fractional-order differential equations and modified projective synchronization of chaotic systems are established. Section 3 presents dynamics and parameters of two kind of fractional-order chaotic systems with time delay, which we use in our simulations. Three examples are provided to validate the theoretical results and to illustrate the application of the proposed strategy in Section 4. Finally, the main conclusions of the work are presented in Section 5.

2. Preliminaries

2.1. Fractional differential operator definitions

To discuss fractional chaotic systems, we usually need to solve fractional-order differential equations. For solving the fractional differential equations fractional operator defined in three more commonly definitions: Grünwald–Letnikov (GL) definition, Riemann–Liouville (RL) definition and Caputo definition. The GL definition of non-integer is:

$$D_t^{\alpha} f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{(t-a)/h} (-1)^j \binom{\alpha}{j} f(t-jh) \tag{1}$$

where $\binom{\alpha}{j} = \frac{\alpha(\alpha-1)\cdots(\alpha-j+1)}{j!}$

Captuo definition is given as below:

$$\frac{d^{\alpha}}{dt^{\alpha}} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{n-\alpha+1}} d\tau \tag{2}$$

The best-known RL definition of fractional-order, which is described by:

$$\frac{d^{\alpha}}{dt^{\alpha}} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^{\alpha}}{dt^{\alpha}} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{n-\alpha+1}} d\tau \tag{3}$$

where n is an integer such that $n-1 < \alpha < n$, $\Gamma(.)$ is the Gamma function.

For the function $f(t)$ having m-order continuous derivatives with $t \geq 0$, the Laplace transform of the Caputo fractional derivative is given as

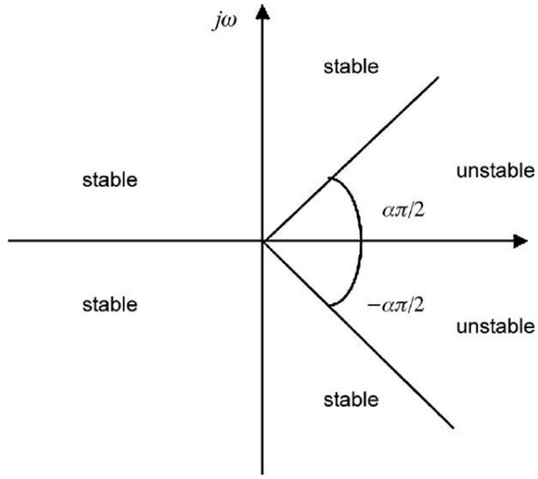

Fig. 1. Stability region of the fractional-order system (5).

$$L[D^{\alpha} f(t), s] = s^{\alpha} F(s) - \sum_{k=0}^{m-1} s^{\alpha-k-1} f(k)(0) \tag{4}$$

where $F(s)$ is the Laplace transform of $f(t)$, and $m = \lceil \alpha \rceil$, i.e., m is the first integer which is not less than α. Obviously, the initial conditions for the fractional differential equation with Caputo derivative are in the same form as those for the integer-order differential equation which have real physical meanings.

2.2. Stability theorem of fractional-order systems

2.2.1. General stability theorem of fractional-order systems

We review some important results of the stability theorems for fractional-order systems here. Consider the following linear system of fractional differential equation [11]

$$D^{\alpha} x = Ax, \quad x(0) = x_0 \tag{5}$$

where $x \in R^n$, $A \in R^{n\times n}$, $\alpha = [\alpha_1, \alpha_2, \cdots, \alpha_i, \cdots, \alpha_n]^T$, $(0 < \alpha_i \leq 1)$ for $(i = 1, 2, \cdots, n)$ indicates the fractional orders.

Case 1. When $\alpha_1 = \alpha_2 = \cdots = \alpha_n = \alpha$, then the autonomous fractional-order system (5) is commensurate and is asymptotically stable if $|arg(spec(A))| > \alpha\pi/2$. In this case the components of the state decay towards 0 like $t^{-\alpha}$. In case of $\alpha = 1$, the above stability agrees with the well-known results for ordinary linear differential systems. Fig. 1 shows the stability region of above fractional-order system.

Case 2. When $\alpha_i s$ for $i = 1, 2, \cdots, n$ are different, then the autonomous fractional-order system (5) is in-commensurate and suppose that $\alpha_i$are rational numbers between 0 and 1. Let M is the least common multiple of the dominator u_i of $\alpha_i' s$, where $i = v_i/u_i$, $rem(u_i, v_i) = 1$, $u_i, v_i \in N$, for $i = 1, 2, \cdots, n$. Then system (5) is asymptotically stable if all the roots of the equation $det(diag(M1, M2, \cdots, Mn)A) = 0$ satisfy $|arg(\lambda)| > \pi/2M$. In what follows, we will use the above stability theorem to choose sufficient control parameters for the synchronization between different fractional-order systems.

2.2.2. Stability condition for fractional differential equations with time delays

For below n-dimensional linear fractional differential system with multiple time delays, the stability condition