Chaos, Solitons and Fractals 78 (2015) 148–155

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

A new Jacobi elliptic function expansion method for solving a nonlinear PDE describing the nonlinear low-pass electrical lines

E.M.E. Zayed*, K.A.E. Alurrfi

Department of Mathematics, Faculty of Science, Zagazig University, P.O. Box 44519, Zagazig, Egypt

ARTICLE INFO

Article history:
Received 28 June 2015
Accepted 16 July 2015
Available online 1 September 2015

Keywords:
New Jacobi elliptic function expansion method
Exact solutions
Nonlinear low-pass electrical lines
Kirchhoff's laws

ABSTRACT

The first elliptic function equation is used in this article to find a new kind of solutions of nonlinear partial differential equations (PDEs) based on the homogeneous balance method, the Jacobi elliptic expansion method and the auxiliary equation method. New exact solutions to the Jacobi elliptic functions of a nonlinear PDE describing the nonlinear low-pass electrical lines are obtained with the aid of computer algebraic system Maple. Based on Kirchhoff's law, the given nonlinear PDE has been derived and can be reduced to a nonlinear ordinary differential equation (ODE) using a simple transformation. The given method in this article is straightforward and concise, and it can also be applied to other nonlinear PDEs in mathematical physics. Further results may be obtained.

© 2015 Elsevier Ltd. All rights reserved.

1. Introduction

In the recent years, investigations of exact solutions to nonlinear PDEs play an important role in the study of nonlinear physical phenomena in such as fluid mechanics, hydrodynamics, optics, plasma physics, solid state physics, biology and so on. Several methods for finding the exact solutions to nonlinear equations in mathematical physics have been presented, such as the inverse scattering method [1], the Hirota bilinear transform method [2], the truncated Painlevé expansion method [3–6], the Bäcklund transform method [7,8], the exp-function method [9–11], the tanh-function method [12,13], the Jacobi elliptic function expansion method [14–16], the $(\frac{G'}{G})$-expansion method [17–22], the modified $(\frac{G'}{G})$-expansion method [23], the $(\frac{G'}{G}, \frac{1}{G})$-expansion method [24–27], the modified simple equation method [28–30], the multiple exp-function algorithm method [31,32], the transformed rational function method [33], the local fractional series expansion method [34], the first integral method [35,36], the generalized Riccati equation mapping method [37,38] and so on.

The objective of this article is to use a new Jacobi elliptic function expansion method to construct the exact solutions of the following nonlinear PDE governing wave propagation in nonlinear low-pass electrical transmission lines [39]:

$$\frac{\partial^2 V(x,t)}{\partial t^2} - \alpha \frac{\partial^2 V^2(x,t)}{\partial t^2} + \beta \frac{\partial^2 V^3(x,t)}{\partial t^2} - \delta^2 \frac{\partial^2 V(x,t)}{\partial x^2} - \frac{\delta^4}{12} \frac{\partial^4 V(x,t)}{\partial x^4} = 0, \tag{1.1}$$

where α, β and δ are constants, while $V(x, t)$ is the voltage in the transmission lines. The variable x is interpreted as the propagation distance and t is the slow time. The physical details of the derivation of Eq. (1.1) using the Kirchhoff's laws are given in [39], which are omitted here for simplicity. Note that Eq. (1.1) has been discussed in [39] using an auxiliary equation method [40] and its exact solutions have been found.

This paper is organized as follows: In Section 2, the description of a new Jacobi elliptic function expansion method is given. In Section 3, we use the given method described in Section 2, to find exact solutions of Eq. (1.1). In Section 4, we solve Eq. (1.1) using a direct method. In Section 5, physical

* Corresponding author.
E-mail addresses: e.m.e.zayed@hotmail.com (E.M.E. Zayed), alurrfi@yahoo.com (K.A.E. Alurrfi).

http://dx.doi.org/10.1016/j.chaos.2015.07.018
0960-0779/© 2015 Elsevier Ltd. All rights reserved.

explanations of some results are presented. In Section 6, some conclusions are obtained.

2. Description of a new Jacobi elliptic function expansion method

Consider a nonlinear PDE in the form

$$P(V, V_x, V_t, V_{xx}, V_{tt}, \dots) = 0, \tag{2.1}$$

where $V = V(x, t)$ is a unknown function, P is a polynomial in $V(x, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. Let us now give the main steps of the Jacobi elliptic function expansion method [41]:

Step 1. We look for the voltage $V(x, t)$ in the traveling form

$$V(x,t) = V(\xi), \qquad \xi = \sqrt{k}(x - \lambda t), \tag{2.2}$$

where k and λ are undetermined positive parameters, and λ is the velocity of propagation, to reduce Eq. (2.1) to the following nonlinear ordinary differential equation (ODE):

$$H(V, V', V'', \dots) = 0, \tag{2.3}$$

where H is a polynomial of $V(\xi)$ and its total derivatives $V'(\xi), V''(\xi), \dots$ and $' = \frac{d}{d\xi}$.

Step 2. We suppose that the solution of Eq. (2.3) has the form:

$$V(\xi) = g_0 + \sum_{i=1}^{N}\left[\frac{z(\xi)}{1+z^2(\xi)}\right]^{i-1}\left\{g_i\left(\frac{z(\xi)}{1+z^2(\xi)}\right) + f_i\left(\frac{1-z^2(\xi)}{1+z^2(\xi)}\right)\right\}, \tag{2.4}$$

where $z(\xi)$ satisfies the Jacobi elliptic equation:

$$\left(z'(\xi)\right)^2 = a + bz^2(\xi) + cz^4(\xi), \tag{2.5}$$

where a, b, c, g_0, g_i, f_i $(i = 1, 2, \dots, N)$ are constants to be determined later, such that $g_N \neq 0$ or $f_N \neq 0$.

Step 3. We determine the positive integer N in (2.4) by balancing the highest-order derivatives and the nonlinear terms in Eq. (2.3).

Step 4. Substituting (2.4) along with Eq. (2.5) into Eq. (2.3) and collecting all the coefficients of $z^i(\xi)$ $(i = 0, 1, 2, \dots)$, then setting these coefficients to zero, yield a set of algebraic equations, which can be solved by using the Maple or Mathematica to find the values of $g_0, g_i, f_i, \lambda, k, a, b, c$.

Step 5. It is well-known [41] that Eq. (2.5) has families of Jacobi elliptic function solutions as follows:

No.	a	b	c	$z(\xi)$
1	1	$-(1+m^2)$	m^2	$sn\xi$
2	$1-m^2$	$2m^2-1$	$-m^2$	$cn\xi$
3	m^2	$-(1+m^2)$	1	$ns\xi = (sn\xi)^{-1}$
4	$-m^2$	$2m^2-1$	$1-m^2$	$nc\xi = (cn\xi)^{-1}$
5	$\frac{1}{4}$	$\frac{1-2m^2}{2}$	$\frac{1}{4}$	$ns\xi \pm cs\xi$
6	$\frac{1-m^2}{4}$	$\frac{1+m^2}{2}$	$\frac{1-m^2}{4}$	$nc\xi \pm sc\xi$ or $\frac{cn\xi}{1\pm sn\xi}$

Note that there are other Jacobi elliptic function solutions of Eq. (2.5) which are omitted here for simplicity.

In this table, $sn\xi = sn(\xi, m)$, $cn\xi = cn(\xi, m)$, $dn\xi = dn(\xi, m)$, $ns\xi = ns(\xi, m)$, $cs\xi = cs(\xi, m)$, $ds\xi = ds(\xi, m)$, $sc\xi = sc(\xi, m)$, $sd\xi = sd(\xi, m)$ are the Jacobi elliptic function with modulus m, where $0 < m < 1$. These functions degenerate into hyperbolic functions when $m \to 1$ as follows: $sn\xi \to \tanh\xi$, $cn\xi \to \mathrm{sech}\xi$, $dn\xi \to \mathrm{sech}\xi$, $ns\xi = \coth\xi$, $cs\xi = \mathrm{csch}\xi$, $ds\xi = \mathrm{csch}\xi$, $sc\xi = \sinh\xi$, $sd\xi = \sinh\xi$, $nc\xi = \cosh\xi$. and into trigonometric functions when $m \to 0$ as follows: $sn\xi \to \sin\xi$, $cn\xi \to \cos\xi$, $dn\xi \to 1$, $ns\xi \to \csc\xi$, $cs\xi \to \cot\xi$, $ds\xi \to \csc\xi$, $sc\xi \to \tan\xi$, $sd\xi \to \sin\xi$, $nc\xi \to \sec\xi$. Also, these functions satisfy the following formulas:

$sn^2\xi + cn^2\xi = 1$, $dn^2\xi + m^2sn^2\xi = 1$, and $sn'\xi = cn\xi dn\xi$, $cn'\xi = -sn\xi dn\xi$, $dn'\xi = -m^2 sn\xi cn\xi$, $cd'\xi = -(1-m^2)sd\xi nd\xi$, $ns'\xi = -cs\xi ds\xi$, $dc'\xi = (1-m^2)nc\xi sc\xi$, $nc'\xi = sc\xi dc\xi$, $nd'\xi = m^2 cd\xi sd\xi$, $sc'\xi = dc\xi nc\xi$, $cs'\xi = -ns\xi ds\xi$, $ds'\xi = -cs\xi ns\xi$, $sd'\xi = nd\xi cd\xi$, where $' = \frac{d}{d\xi}$.

Step 6. Substituting the values of $g_0, g_i, f_i, k, \lambda, a, b, c$ as well as the solutions of Eq. (2.5) obtained in Step 5, into (2.4) we have the exact solutions of Eq. (2.1).

3. Exact solutions of Eq. (1.1) using the proposed method of Section 2

In this section, we apply the Jacobi elliptic function expansion method of Section 2 to find families of new Jacobi elliptic function solutions of Eq. (1.1). To this end, we use the transformation (2.2) to reduce Eq. (1.1) to the following nonlinear ODE:

$$\frac{d^2}{d\xi^2}\left\{\frac{k^2\delta^4}{12}\frac{d^2V}{d\xi^2} + (k\delta^2 - k\lambda^2)V + \alpha k\lambda^2V^2 - \beta k\lambda^2V^3\right\} = 0. \tag{3.1}$$

Integrating Eq. (3.1) twice and vanishing the constants of integration, we find the following ODE:

$$\frac{K^2}{12}\frac{d^2V}{d\xi^2} + (K - U)V + \alpha UV^2 - \beta UV^3 = 0. \tag{3.2}$$

where $K = k\delta^2$ and $U = k\lambda^2$.

Balancing $\frac{d^2V}{d\xi^2}$ with V^3 gives $N = 1$. Therefore, (2.4) reduces to

$$V(\xi) = g_0 + g_1\left(\frac{z(\xi)}{1+z^2(\xi)}\right) + f_1\left(\frac{1-z^2(\xi)}{1+z^2(\xi)}\right), \tag{3.3}$$

where g_0, g_1 and f_1 are constants to be determined such that $g_1 \neq 0$ or $f_1 \neq 0$.

Substituting (3.3) along with Eq. (2.5) into Eq. (3.2) and collecting all the coefficients of $z^i(\xi)$, $(i = 0, 1, \dots, 6)$ and setting them to zero, we have the following algebraic equations:

$$z^6 : 4cK^2f_1 - 12Kf_1 + 12Kg_0 + 12U\beta f_1^3 - 36U\beta f_1^2 g_0 + 12U\alpha f_1^2 + 36U\beta f_1 g_0^2 - 24U\alpha f_1 g_0 + 12Uf_1 - 12U\beta g_0^3 + 12U\alpha g_0^2 - 12Ug_0 = 0,$$

$$z^5 : 12Kg_1 - 12Ug_1 + K^2bg_1 - 6K^2cg_1 - 36U\beta f_1^2 g_1 - 36U\beta g_0^2 g_1 - 24U\alpha f_1 g_1 + 24U\alpha g_0 g_1 + 72U\beta f_1 g_0 g_1 = 0,$$

$$z^4 : 36Kg_0 - 12Kf_1 + 12Uf_1 - 36Ug_0 + 8K^2bf_1 - 12K^2cf_1$$

Download English Version:

https://daneshyari.com/en/article/1888532

Download Persian Version:

https://daneshyari.com/article/1888532

Daneshyari.com