Chaos, Solitons and Fractals 78 (2015) 156–161

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Transition from wave pattern to stationary pattern in a spatial predator-prey system with delay

Caiyun Wang*, Lizhi Wang

Department of Mathematics, Xinzhou Teachers University, Xinzhou, Shanxi 034000, People's Republic of China

ARTICLE INFO

Article history:
Received 30 May 2015
Accepted 20 July 2015
Available online 1 September 2015

PACS:
87.23.Cc
82.40.Ck
82.40.Bj
05.45.Pq

Keywords:
Predator-prey
Time delay
Transition
Stationary pattern

ABSTRACT

Spatial diffusion and time delay are two main important factors in biological systems. In this paper, a predator-prey model with Holling III functional response, which includes time delay and diffusion processes is presented. It was found that time delay can induce transition from wave pattern to stationary pattern. Furthermore, for different values of time delay, different types of stationary patterns are obtained in the predator-prey model. These results may be useful for us to understand the pattern transition arising from intrinsic elements in the real ecosystems.

1. Introduction

Predation behavior is one of the most important phenomenon in ecosystems, which can cause the recycle of materials. And hence predator-prey models are widely investigated by both mathematicians and ecologists [1]. Many investigations have revealed that spatial distribution of nutrients as well as interactions on spatial scales have an important impact on the dynamics of ecological populations [2–6]. In order to know the spatial effects well, many scholars pay more and more attention to the spatial predator-prey systems, especially the pattern formation [[7–28].

On other hand, predator-prey systems with time delays have been of considerable interest. It means the time between immature and mature [29], the time of pregnancy of the mother [30] and so on [31–34]. Time delay plays a significant role in predator-prey systems in three main aspects. First, it changes the dynamical behavior from stable state to unstable state. Second, it may result in the appearance of bifurcation behavior including Hopf bifurcation and so on. Third, it may induce the emergence of coexistence of multiple attractors or chaos.

There has been some work on spatial predator-prey models with time delay. Wang et al. presented a Holling–Tanner model with delay and found that if value of delay is more than a critical value, the model will have periodic solutions [35]. Tang and Song studied a delayed diffusive predator-prey model with herd behavior and obtained Hopf bifurcation by using normal form theory and the center manifold argument for partial functional differential equations [36]. Banerjee and Zhang showed the influence of discrete delay on pattern formation in a ratio-dependent predator-prey model [37]. However, how time delay has effects on pattern transition, especially from non-stationary pattern to stationary pattern is not well understood. For such reason, we will investigate the influence of time delay on pattern transition in a predator-prey model in this paper.

* Corresponding author. Tel.: +86 15676544568.
E-mail address: xzwcy1234@126.com (C. Wang).

http://dx.doi.org/10.1016/j.chaos.2015.07.021
0960-0779/

This paper is organized as follows. In Section 2, we give a spatial Holling III model and interpret the biological meanings of these parameters of the model. In Section 3, we show the emergence of pattern transition from wave pattern to stationary pattern. What is more, different types of stationary patterns are obtained. In the last section, some conclusions and discussion are given.

2. A spatial predator-prey system with delay

Feeding rate of predator upon prey is one of the most significant components of the predator-prey relationship, i.e., the predator's functional response. In this paper, we mainly focus our attention on a predator-prey system with Holling-III functional response [38,39]:

$$\frac{dn}{dt} = rn\left(1 - \frac{n}{K}\right) - \frac{an^2 p}{b + n^2}, \tag{1a}$$

$$\frac{dp}{dt} = \frac{ean^2 p}{b + n^2} - dp, \tag{1b}$$

where n and p denote the prey and predator populations respectively. r represents the intrinsic growth rate of the prey, K is the carrying capacity of the prey in the absence of predator and harvesting, e is the conversion factor denoting the number of newly born predators for each captured prey and d is the death rate of the predator. The term $an^2/(b + n^2)$ denotes the functional response of the predator which is termed as a Holling-III response function.

In order to minimize the number of parameters involved in the model system it is extremely useful to write the system in non-dimensionalized form. Although there is no unique method of doing this, it is often a good idea to relate the variables to some key relevant parameters. Thus, in (1), we take

$$N = \frac{n}{\sqrt{b}}, \quad P = \frac{p}{\sqrt{b}}, \quad \bar{t} = \frac{rt\sqrt{b}}{K},$$

$$\alpha = \frac{K}{\sqrt{b}}, \quad \beta = \frac{aK}{r\sqrt{b}}, \quad \gamma = \frac{eaK}{r\sqrt{b}}, \quad \mu = \frac{dK}{r\sqrt{b}}.$$

This leads to (after dropping the bar) the nondimensional system:

$$\frac{dN}{dt} = N(\alpha - N) - \frac{\beta N^2 P}{1 + N^2}, \tag{2a}$$

$$\frac{dP}{dt} = \frac{\gamma N^2 P}{1 + N^2} - \mu P. \tag{2b}$$

When combined with spatial factor and time delay, the original spatially extended system is written as the following system:

$$\frac{\partial N(\vec{r},t)}{\partial t} = N(\vec{r},t)[\alpha - N(\vec{r},t)] - \frac{\beta N(\vec{r},t)^2 P(\vec{r},t)}{1 + N(\vec{r},t)^2} + D_1 \nabla^2 N(\vec{r},t), \tag{3a}$$

$$\frac{\partial P(\vec{r},t)}{\partial t} = \frac{\gamma N(\vec{r},t-\tau)^2 P(\vec{r},t)}{1 + N(\vec{r},t-\tau)^2} - \mu P(\vec{r},t) + D_2 \nabla^2 P(\vec{r},t), \tag{3b}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the usual Laplacian operator in two-dimensional space. The diffusion coefficients are denoted by D_1 and D_2, respectively.

3. Mathematical analysis

Before proceeding to the spatially explicited case, the first step is to check the properties of the local dynamics, i.e., the properties of the Eq. (2). Eq. (2) has the following stationary steady:

(i) $E_0 = (0, 0)$ and $E_1 = (\alpha, 0)$, corresponding to the extinction of the predator;
(ii) interior equilibrium point $E^* = (N^*, P^*)$, corresponding to coexistence of prey and predator, where

$$N^* = \frac{\sqrt{\mu(\gamma - \mu)}}{\gamma - \mu}, \tag{4a}$$

$$P^* = \frac{\gamma(\alpha - N^*)}{\beta N^*(\gamma - \mu)}. \tag{4b}$$

The condition to ensure the existence of interior equilibrium point is $\mu < \gamma < \mu(1 + \alpha^2)/\alpha^2$.

Assume that the Jacobian matrix of system (3) at interior equilibrium point is

$$J = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}.$$

Let $u = N - N^*$ and $v = P - P^*$ be spatial perturbation at the equilibrium (N^*, P^*), then linearizing the reaction-diffusion system (3) at (N^*, P^*) yields

$$\frac{\partial u(\vec{r},t)}{\partial t} = a_{11}u(\vec{r},t) + a_{12}v(\vec{r},t) + D_1\nabla^2 u(\vec{r},t), \tag{5a}$$

$$\frac{\partial v(\vec{r},t)}{\partial t} = a_{21}u(\vec{r},t-\tau) + a_{22}v(\vec{r},t) + D_2\nabla^2 v(\vec{r},t). \tag{5b}$$

We want to examine the linear stability of (N^*, P^*) of system (3) by linearizing the dynamic system (3) around the spatially homogeneous fixed point (0, 0) for small space- and time-dependent fluctuations and expand them in Fourier space [40–42]:

$$(u, v)^T = (A_1, A_2)^T e^{\lambda t + i\kappa \vec{r}}, \tag{6}$$

which yields

$$A_1 \lambda e^{\lambda t + i\kappa \vec{r}} = (A_1 a_{11} + A_2 a_{12} - A_1 D_1 \kappa^2) e^{\lambda t + i\kappa \vec{r}}, \tag{7a}$$

$$A_2 \lambda e^{\lambda t + i\kappa \vec{r}} = (A_1 a_{21} e^{-\lambda\tau} + A_2 a_{22} - A_2 D_2 \kappa^2) e^{\lambda t + i\kappa r}. \tag{7b}$$

Because that $e^{\lambda t + i\kappa \vec{r}} \neq 0$, Eq. (7) is equivalent to the following linear algebraic equations:

$$\begin{pmatrix} \lambda - a_{11} + D_1\kappa^2 & -a_{12} \\ -a_{21}e^{-\lambda\tau} & \lambda - a_{22} + D_2\kappa^2 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{8}$$

The condition of the existence of nontrivial solutions in Eq. (8) is:

$$\mathrm{Det}\begin{pmatrix} \lambda - a_{11} + D_1\kappa^2 & -a_{12} \\ -a_{21}e^{-\lambda\tau} & \lambda - a_{22} + D_2\kappa^2 \end{pmatrix} = 0. \tag{9}$$