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Stochastic stability of stochastic switched epidemic models with constant and impulsive control schemes

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ABSTRACT

This paper investigates stochastic stability for stochastic switched AIDS (Acquired Immune Deficiency Syndrome) models with constant and impulsive control schemes. The stochasticity is introduced via the technique of parameter perturbation and the switching is assumed that the models parameters are time-varying functions and switch their forms in time. First, a stochastic switched AIDS model with constant control schemes is studied, and new sufficient conditions are established by using the Lyapunov–Razumikhin method. The results show that the system is stable under the condition $\bar{R} < 1$, regardless of whether the subsystems are unstable or stable, which implies that the disease could be eradicated theoretically. Furthermore, impulsive control schemes are applied into a stochastic switched AIDS model. Threshold conditions on the basic reproduction number are developed which guarantee the system is stochastically stable. In addition, complex dynamic behavior for the positive periodic solution is analyzed, and the results imply that less vaccination could lead theoretically the disease to die out. Numerical examples are employed to verify the main results.

1. Introduction

The AIDS epidemic is still a global challenge since the first patients were recognized in 1981. Viral transmission occurs through direct contact of a mucous membrane or the bloodstream with a bodily fluid containing HIV (Human Immunodeficiency Virus), such as blood, preseminal fluid, and breast milk [1]. It is reported that 35 million individuals (including 3.2 million children) lived with the disease worldwide until the end of the year 2013, and 2.1 million people became newly infected and 1.5 million died of AIDS-related illnesses in the same year [2]. Thus, it has become a major issue that how to effectively prevent or control epidemics as they spread through population.

Mathematical models of AIDS epidemic have played an important role in understanding the disease dynamics and controlling its spread. Mukandavire et al. [3] presented an HIV/AIDS model with explicit incubation period, and investigated the local stability of the disease-free and endemic equilibria. Okosun et al. [4] studied the impact of optimal control on the treatment of HIV/AIDS, and derived the conditions for optimal control of the disease with condoms, treatment regime and screening of infections. Effective strategies have been proposed to control or eradiate infectious disease such as hepatitis B and measles [5]. The standard conventional approach is a continuous constant vaccination, which has been considered in many literatures [6,7]. It is predicted that conventional vaccination strategies can clear the disease if the proportion of the successfully vaccinated individual is higher than a certain critical value (around 95% for measles) [8]. Recently, another method, impulsive control strategies has gained prominence due to its highly successful applications in the control of viral infections such as hepatitis B, Measles and Smallpox [9]. At each vaccination time, a fraction of the population is vaccinated, who will

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obtain a permanent or temporary immunity. This kind of vaccination is called impulsive, since all the vaccine doses are applied in a time which is very short with respect to the dynamics of disease. Pulse vaccination strategy was firstly studied by Agur et al. [10]. From then on, there have been numerous studies on pulse vaccination in epidemic models.

Experimental evidence shows that the infectious diseases are inevitably influenced by environmental factors such as temperature, and the immunological state of the host. In some sense, stochastic dynamical models give better compatibility with reality as they provide some additional degree of realism compared to their deterministic counterpart [11–14]. Incorporation of stochasticity in epidemic models has received increasing attention in recent years. Hao et al. [15] studied a stochastic tumor growth model with immunization.

On the other hand, models coefficients in the epidemic literature (for example, see [16]) are assumed to be unchanged during the epidemic spreading. In fact, many infectious diseases fluctuate over time. For example, the weather changes may lead to the variation of the disease spreading. The human behavior changes affect the epidemic dynamics [17]. Some authors [18,19] also modeled the term-time forcing epidemic models, where the models' parameters change abruptly in time (for instance, due to holiday breaks for school children). It is more realistic to introduce switching into epidemic models in some cases [[20,21,22].

The approach in this paper is to formulate stochastic switched AIDS models as stochastic switched systems. More specifically, the random factors in the form of Gaussian white noise are introduced into a deterministic AIDS epidemic model; the model parameters are considered to be switching parameters and switch their functional forms in time due to the underlying mechanics governing the spread of a disease which change in time. The basic AIDS model can be extended as a stochastic switched HIV model. Switched systems are special class of hybrid control systems, which are governed by a combination of continuous and discontinuous subsystems and a switching rule. One main feature of the switched system is that the included switching rule may induce stability of the switched system composed of unstable subsystems [23,24]. The objective of this paper is to use this approach to investigate the application of vaccination control schemes (in the event a potential effective vaccine is developed) to the stochastic switched AIDS models. Both constant and impulsive control schemes are examined to analyze their success in eradicating the disease by using some of the switched systems techniques [25]. To the best of the authors' knowledge, no research has been done about the stochastic AIDS models with switching parameters and control schemes. Therefore, the research on the stochastic switched AIDS dynamical models with constant and impulsive control strategies is very important.

The paper is organized as follows: In Section 2, a constant control scheme is applied to stochastic AIDS models with switching parameters. Threshold criteria are established to examine the disease extinction or persistence from the stochastic switched dynamical point of view. In Section 3, impulsive control schemes are introduced into stochastic AIDS models with switching parameters. Sufficient conditions for the control scheme model are established on the basic reproduction number. Numerical simulations are given to illustrate our results in Section 4. Some conclusions and perspectives are given in Section 5.

2. Constant control schemes

Consider the population size is divided into five classes, namely susceptible population $S(t)$, asymptomatic infective population $I_1(t)$ (showing no symptoms of the disease), symptomatic infective population $I_2(t)$ (showing symptoms of the disease), the full blown AIDS population $A(t)$, and the vaccinated population $U(t)$. The susceptible population may be infected via shared injections or sexual contacts with two types of infective population. The full blown AIDS population and the vaccinated population are assumed not to be transmitting the disease.

The model parameters (such as death rate and the contact rate) are always affected by the random fluctuating environment. Stochasticity is introduced into an AIDS model via the technique of parameter perturbation which is standard in stochastic population modeling. Moreover, due to the seasonal variety, biological and environmental parameters are naturally subjected to fluctuation in time. To investigate this kind of problems, assume that the coefficients of the model are time-varying and switching in time. Assume that these parameters are modeled as switching parameters and are governed by a switching rule $\sigma(t) : (t_{k-1}, t_k] \to \{1, 2, \dots, m\}, k = 1, 2, \dots,$ which is a piecewise continuous (from the left) function of time, where m is the number of the subsystem. The switching time $\{t_k\}$ satisfies $t_k > t_{k-1}$ and $t_k \to \infty$ as $k \to \infty$. Denote the set of all switching rules by $\mathcal{I}$. A simple periodic switching rule (1) is given, and Fig. 1 is its illustration.

$$\sigma(t) = \begin{cases} 1, & \text{if} \quad t \in (k, k+0.5], \\ 2, & \text{if} \quad t \in (k+0.5, k+1], \end{cases} \quad k = 0, 1, 2, \dots \tag{1}$$

Assume that a fraction τ of asymptomatic infective population is treated continuously in time and moved to the vaccinated class. This leads to a new stochastic switched AIDS epidemic model with constant control scheme,

$$\begin{cases} dS = (\lambda - \frac{p_\sigma SI_1}{N} - \frac{q_\sigma SI_2}{N} - uS)dt - \varphi_1 SdW(t), \\ dI_1 = (\frac{p_\sigma SI_1}{N} + \frac{q_\sigma SI_2}{N} - (r_\sigma + a_\sigma + u)I_1 - \tau I_1)dt \\ \qquad - \varphi_2 I_1 dW(t), \\ dI_2 = (e_\sigma r_\sigma I_1 - (b_\sigma + u)I_2)dt - \varphi_3 I_2 dW(t), \\ dA = ((1-e_\sigma)r_\sigma I_1 + b_\sigma I_2 - (c_\sigma + u)A)dt - \varphi_4 AdW(t), \\ dU = (-uU + \tau I_1)dt, \end{cases} \tag{2}$$

where λ is the recruitment rate of susceptible population; u is the natural death rate of all human classes; the switching function p_σ is the contact rate between susceptible and asymptomatic infective population; q_σ is the contact rate between susceptible and symptomatic infective population; $e_\sigma r_\sigma$ is the rate at which asymptomatic infective population are detected by a screening method to become symptomatic infective population; $(1 - e_\sigma)r_\sigma$ is the rate at which asymptomatic infective population become the full blown AIDS population. a_σ, b_σ, and c_σ are disease-caused death