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Effect of magnetic field on the radial pulsations of a gas bubble in a non-Newtonian fluid

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ABSTRACT

Dynamics of acoustically driven bubbles' radial oscillations in viscoelastic fluids are known as complex and uncontrollable phenomenon indicative of highly active nonlinear as well as chaotic behavior. In the present paper, the effect of magnetic fields on the non-linear behavior of bubble growth under the excitation of an acoustic pressure pulse in non-Newtonian fluid domain has been investigated. The constitutive equation [Upper-Convective Maxwell (UCM)] was used for modeling the rheological behaviors of the fluid. Due to the importance of the bubble in the medical applications such as drug, protein or gene delivery, blood is assumed to be the reference fluid. It was found that the magnetic field parameter (B) can be used for controlling the nonlinear radial oscillations of a spherical, acoustically forced gas bubble in nonlinear viscoelastic media. The relevance and importance of this control method to biomedical ultrasound applications were highlighted. We have studied the dynamic behavior of the radial response of the bubble before and after applying the magnetic field using Lyapunov exponent spectra, bifurcation diagrams and time series. A period-doubling bifurcation structure was predicted to occur for certain values of the parameters effects. Results indicated its strong impact on reducing the chaotic radial oscillations to regular ones.

1. Introduction

The dynamics of bubble formation and collapse have been studied using a number of publications, including the studies of radial oscillating bubbles by Rayleigh [1], Plesset [2,3], Crum et al. [4], Flynn [5], Lauterborn [6], Plesset et al. [7], Prosperetti [8–10] and so on. Therefore, it is important to develop a technique in order to study the bubble radial stability in distinctive situations. In view of the escalating use of the bubbles in new applications, particularly medical and industrial, the number of studies on the growth and collapse of the bubbles in different structures and environments has increased [11]. In more important medical applications, bubbles are used for the delivery of drugs [12–14], cancer treatment [15–17], and the barrier opening of clogged veins and arteries [18,19]. In all cases, bubbles should move and grow in the blood stream and collapse in the intended location. So it is important to take the bubbles radius motion stable and not permit to collapse until the required region. The research conducted on blood indicates that approximation blood rheology by non-Newtonian models, correlates well with the experimental results [20,21].

Therefore, the study of bubble growth and its stability in non-Newtonian fluid will be of the most important concern [22]. The chaotic behavior of bubbles moving in a

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non-Newtonian fluid has been investigated experimentally by Jiang et al. [23]. In addition to experimental studies [24–27], there have also been many theoretical investigations on bubble growth [28–32]. In the article presented by Wang et al. [33], the nonlinear vibration of a protein bubble submerged in bingham liquid has been mathematically modeled, and the bubble's reaction to pressure pulses has been studied. By presenting an analytical model for bubble growth in linear viscoelastic fluids and solving it through the perturbation method, Allen et al. [34] showed that the increase in the Deborah number leads to an increase in bubble radial oscillation amplitude. Deborah number is a non-dimensional elastic parameter which is defined as the ratio of the relaxation time and characteristic timescale for the bubble radius oscillation [34]. In another article, Allen et al. [35] extended his analytical model to nonlinear, non-Newtonian fluid (UCM fluid) and used numerical methods to solve the integro-differential equations. They have also demonstrated the increase in bubble radial oscillation amplitude with the increase in the Deborah number. In the work of Jimenez-Fernandez [36], through the development of analytical relations for bubble growth in non-Newtonian fluid fields affected by the external pulses, the growth of bubbles under the influence of factors like pulse intensity, the Reynolds number and the amount of elasticity has been investigated. In this study, it has been emphasized that with the increase in the Deborah number, bubble growth will become chaotic, and the bubble will approach the state of collapse.

Furthermore, in different theoretical studies, the subject of bubble growth in non-Newtonian fluid showed that in cases where the Reynolds number is of the order 1, the growth and collapse of bubbles can be controlled via Newtonian viscosity. Lind and Phillips [37] have demonstrated the growth of bubbles in non-Newtonian fluids through different constitutive equations. According to their results, at large Deborah numbers, a bubble displays a completely elastic behavior, and its energy diagram indicates a rebound in bubble growth. Brujan et al. [38] used the perturbation method to study the growth of bubbles in non-Newtonian compressible fluids. They showed that at larger Reynolds numbers, sound emission plays the major role in the damping of bubble radial oscillations. Also because of the importance of bubble dynamics, several studies have been conducted on the subject of bubble stability. That is, when the bubble motion gets chaotic, its behavior becomes unpredictable and difficult to deal with [39,40]. In this case, the chaotic nature of the equation requires particular tools for resolution because of the inadequacy of the analytical and linear solutions. By using the primary theory of dynamic systems, Bloom [32] has presented the stable and unstable behaviors of bubbles in non-Newtonian fluids. Aliabadi et al. [41] examined the growth of bubbles in a non-Newtonian fluid field. They have demonstrated that the bubble radial oscillation amplitude decreases under the influence of a magnetic field. Building upon Bloom and Aliabadi's work, the enhanced understanding of the behavior of bubbles in non-Newtonian fluids as well as the ability to reduce the chaotic radial oscillations could be the first step in controlling the bubble dynamics.

The main argument of this study focuses on various aspects of the dynamics of bubbles in non-Newtonian fluids with the presence of magnetic fields. In addition, the effects of substantial parameters that influence the bubble dynamics are studied in a large domain using chaos theory and considering the measure of the non-Newtonian state of the fluid (Deborah number). Bifurcation and Lyapunov exponent diagrams [42–44] are presented for special cases to determine the chaotic regions. Comprehensive information is presented about extremely nonlinear pulsations of bubbles in non-Newtonian fluids at high amplitudes of acoustic pressure where deterministic chaos manifests itself in order to determine the stable and chaotic regions of the system, particularly for drug and gene delivery applications where the applied acoustic pressure is considerably greater than the pressure employed in the ultrasound imaging.

It has been shown that by imposing a radial magnetic field, the rate of growth and collapse of the bubbles dampens considerably. Increasing the magnitude of the magnetic field will cause an increase in the damping effect and, as a result, the growth and collapse of the bubbles can be controlled. The effects of magnetic fields, acoustic field properties and the Deborah number on stability of non-Newtonian fluids are discussed in the following sections.

2. Dynamics of spherical bubble in viscoelastic fluids

The governing equation of bubble growth in non-Newtonian fluid follows the general Rayleigh–Plesset (GRP) equation, and with regards to the viscoelastic effects of the fluid, the following integro-differential equation is obtained [35]:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left[p_g - p_\infty - \frac{2\sigma}{R} + 2\int_R^\infty \left(\frac{\tau_{rr} - \tau_{\theta\theta}}{r}\right) dr\right] \tag{1}$$

In the above equations, τ_{rr} and $\tau_{\theta\theta}$ are components of the shear stress tensor, which have a non-uniform field distribution because of the deformation that exists in the fluid field. Eq. (1) has been written for a bubble with radius R which is affected by a pressure field far away from the bubble, p_∞, in the form of $p_0 + P_a \sin(\omega t)$, where p_0 is the ambient pressure. The pressure pulse enters the fluid field with angular frequency ω and pressure amplitude P_a. Also p_g and σ denote the uniform pressure inside the bubble and surface tension of the fluid, respectively. For simplicity, we assume that the internal gas follows a polytropic relationship with exponent k, and we have $p_g = p_{g_0}(\frac{R_0}{R})^{3k}$, where p_{g_0} and R_0 are the gas bubble pressure and the bubble radius at the initial equilibrium state respectively. By considering the UCM time derivative method [34,35], the radial and theta stress tensor terms will be obtained through the following simplified differential equations:

$$\begin{cases} \tau_{rr} + \lambda_1\left(\frac{\partial \tau_{rr}}{\partial t} + \frac{R^2\dot{R}}{r^2}\frac{\partial \tau_{rr}}{\partial r} + \frac{4R^2\dot{R}}{r^3}\tau_{rr}\right) = 4\eta_0\frac{R^2\dot{R}}{r^3}, \\ \tau_{\theta\theta} + \lambda_1\left(\frac{\partial \tau_{\theta\theta}}{\partial t} + \frac{R^2\dot{R}}{r^2}\frac{\partial \tau_{\theta\theta}}{\partial r} - \frac{2R^2\dot{R}}{r^3}\tau_{\theta\theta}\right) = -2\eta_0\frac{R^2\dot{R}}{r^3}. \end{cases} \tag{2}$$

where η_0 is the zero shear-rate viscosity, λ_1 is the relaxation time, and r is the distance of each element from the coordinate system's origin. By applying the perturbation method, Allen et al. [34,35] solved the above coupled equations and then, in 2001, they introduced the transformation $y = r^3 - R^3(t)$ to immobilize the coordinate by using the Lagrangian