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Ghost responses of the FitzHugh–Nagumo system induced by colored noise

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ABSTRACT

We investigate both numerically and experimentally how the triggering of Ghost Stochastic Resonance is affected by colored noise in a FitzHugh–Nagumo circuit. It is experimentally shown that when the circuit is excited with a bichromatic signal, weak colored noise can induce a response with a main ghost frequency which is not present in the excitation. We analyze the occurrence of this ghost frequency versus the noise intensity and the correlation time of the colored noise. Numerical simulations and experiments confirm that for larger noise correlation time, submultiples of this ghost frequency dominate the system response while the previous expected ghost frequency is less observed.

1. Introduction

Over this century, the properties of nonlinear differential equations have been widely analyzed to provide a better understanding of nature. Indeed, the diversity of the dynamics offered by nonlinear systems constitutes a genuine advantage to describe the complexity of behaviors observed in nature. The excitability of cardiac tissues, the blood pressure propagation in arteries, the electrical activity of neurons, the dynamics of DNA and transport mechanisms in living cells constitute a non restrictive list of behaviors which can be accurately predicted with nonlinear systems [1,2].

Another interesting feature of nonlinear equations is that they arise in many areas of sciences such as optics, chemistry or electrical engineering [3,4]. Vertical cavity surface emitting lasers [5,6], chemical media [7,8] or electronic circuits [9–14] have then attracted a considerable interest to experimentally confirm theoretical or numerical predictions of the response of these nonlinear systems. The development of these experiments has especially allowed to prove the real existence of novel phenomena suspected to exist in numerical simulations [15–19].

Since random fluctuations are omnipresent in nature, scientist efforts have rapidly focused on including the contribution of noise [20]. Novel nonlinear effects have been raised with promising applications in various fields. A common feature shared by these nonlinear stochastic effect is a resonant behavior versus the noise intensity. For instance, Coherence Resonance provides a maximum of regularity in the system response when noise only drives the system [21]. This phenomenon has allowed to explain the activity of neurons in absence of external stimuli [22], mechanisms which involve apnea in clinical diseases states [23]. Experiments have then been carried out to confirm the existence of this nonlinear signature in electronic circuits [24,25], chemical media [26] or optical devices [27,28]. Surprisingly, noise has also revealed the possibility to enhance the stability of the transient dynamics of certain nonlinear systems [29]. This noise enhanced stability effect has been experimentally shown in electronic circuit including tunnel diode [30] and has provided an accurate description of the dynamics of complex nonequilibrium systems which can be found in several scientific fields from chemistry, physics to biology [31,32].

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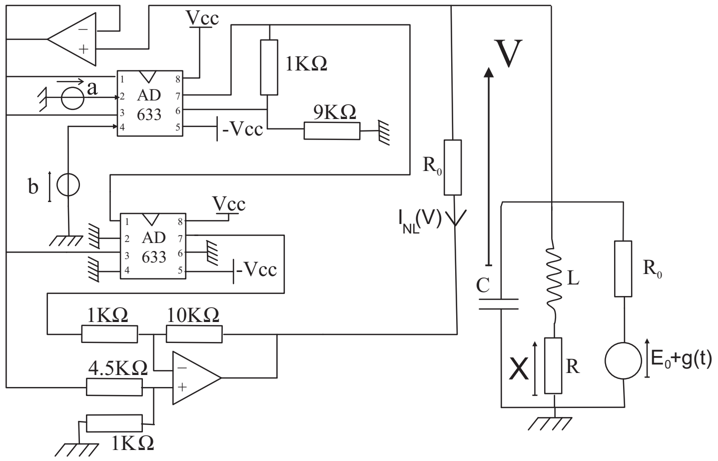

Fig. 1. Sketch of the FitzHugh–Nagumo circuit. AD633 analog multipliers and TL081 amplifiers are used to produce the nonlinear current $I_{NL}(V)$ defined by Eq. (1) and whose current-voltage characteristic is adjusted with the constant voltage sources a and b. A voltage generator provides the excitation $g(t)$ and the offset voltage E_0 which set the FitzHugh–Nagumo circuit in the excitable mode. Component values $R_0 = 1.33$ kΩ, $R = 314$ Ω, $C = 22$ nF and $L = 10.5$ mH. The offset voltages are $E_0 = -1$ V, $a = 2$ V and $b = -2.6$ V while the voltages supply are set to $V_{cc} = 15$ V.

However, the most famous stochastic nonlinear signature is undoubtedly Stochastic Resonance (*S.R.*) which was introduced to explain several phenomena like the ice ages recurrence, the way neurons use noise to detect weak subthreshold stimuli and so on [33–36]. The S.R. effect consists in enhancing the response of a nonlinear system to a coherent signal of information for an optimal noise intensity. It has been widely shown in different experimental systems including, electronic circuits [37–40], chemical media [41,42] or optical systems with potential applications in signal detection [43–46]. A decade ago, a novel form of Stochastic Resonance, called Ghost Stochastic Resonance, has been introduced to explain how the auditory system behaves to perceive two tones of close frequencies [47–49]. In fact, under the driving of a bichromatic signal, that is the sum of two sinusoidal signals of close frequencies, a nonlinear system can use noise to produce a response at a frequency which is not present in the bi-chromatic excitation [50,51]. This ghost frequency can be revealed for an optimal noise intensity leading to the term Ghost Stochastic Resonance. This effect has found numerous applications in different assorted fields ranging from climate dynamics to neuroscience [52–55]. Moreover, an inspection in presence of noise of the regularity of the spike trains response to more specific ratio of the input excitation frequencies has allowed to explain how the auditory system perceives the harmony of musical chords as pleasant or not [56]. Therefore, considering bi-chromatic excitation corrupted by noise in the context of Ghost Stochastic Resonance has been a starting point to better understand mechanisms of sound perception.

However, contrary to other nonlinear stochastic effects where the color of noise have been addressed [57–61], most of the studies devoted to Ghost Stochastic Resonance have been restricted to the contribution of white noise sources [62,63]. The influence of noise features in the ghost responses of these nonlinear systems still remains an open problem.

The aim of this paper is to experimentally analyze how a colored noise can affect the phenomenon of Ghost Stochastic Resonance in a classical model of neuron: the FitzHugh–Nagumo model. To that end, we propose to study the ghost responses of an electronic circuit whose voltage is ruled by the FitzHugh–Nagumo set of equations and which is driven by a bi-chromatic signal corrupted by an Ornstein–Uhlenbeck noise. We also propose to compare our experimental results with those obtained by simulating the FizHugh–Nagumo model. First, we briefly present our circuit. Then, in Section 3, we detail our experimental set-up to observe Ghost Stochastic Resonance. In Section 4, we consider the case of harmonic excitations obtained when the two frequencies f_1 and f_2 which drive the circuit are multiple of the ghost frequency f_0. More precisely, the influence of the noise correlation time is analyzed. Then, Section 5 is devoted to the response of the circuit to inharmonic excitations when different correlation times of the colored noise are considered. The last section is a discussion concluding this paper

2. The nonlinear circuit

Throughout this paper, we perform our experiments with the electronic circuit of Fig. 1 which has already revealed different nonlinear phenomena such as Coherence Resonance or Vibrational Resonance [24,64,65]. The nonlinear current $I_{NL}(V)$ through the resistor R_0 of Fig. 1 allows to implement the nonlinearity requested by the FitzHugh–Nagumo model. Indeed, classical TL081CN operational amplifiers and AD633JNZ analog multipliers are connected to produce a current which obeys to

$$I_{NL}(V) = \frac{V(V-a)(V-b)-V}{R_0}, \tag{1}$$

where a and b are tuned by external continuous voltage sources.

Next, by expressing the current in each branch of the circuit with the Kirchhoff's laws, it can be shown that the dynamics of the voltages V and X are ruled by the following set