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# When does inferring reputation probability countervail temptation in cooperative behaviors for the prisoners' dilemma game?

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ABSTRACT

In evolutionary games, the temptation mechanism reduces cooperation percentage while the reputation mechanism promotes it. Inferring reputation theory proposes that agent's imitating neighbors with the highest reputation takes place with a probability. Although reputation promotes cooperation, when and how it enhances cooperation is still a question. This paper investigates the condition where the inferring reputation probability promotes cooperation. Hence, the effects of reputation and temptation on cooperation are explored under the spatial prisoners' dilemma game, utilizing the methods of simulation and statistical analysis. Results show that temptation reduces cooperation unconditionally while reputation promotes it conditionally, i.e. reputation countervails temptation conditionally. When the inferring reputation probability is less than 0.5, reputation promotes cooperation substantially and thus countervails temptation. However, when the inferring reputation probability is larger than 0.5, its contribution to cooperation is relatively weak and cannot prevent temptation from undermining cooperation. Reputation even decreases cooperation together with temptation when the probability is higher than 0.8. It should be noticed that inferring reputation does not always succeed to countervail temptation and there is a specific interval for it to promote cooperation.



## 1. Introduction

For researchers, promoting cooperation has been a long-lasting pursuit as cooperation brings public good for our society [1–9]. In evolutionary game theories, it has been substantially researched that temptation seduces individuals to defect or cheat on cooperative partners and therefore reduces the cooperation propensity or rate within most groups [1,5,10]. Temptation means that the defect strategy brings more payoffs, which suits Darwinism, and individuals tend to defect other than cooperate [5,6,10]. If no mechanisms counterbalance temptation, cooperation will be doomed.

In order to enhance cooperation, related models or solutions are proposed [1–10]. In evolutionary game theories, the core idea is to design certain mechanisms countervailing temptation that undermines cooperation [6,11–13]. Three models are commonly applied to investigate temptation and possible countervailing mechanisms [14–25], such as prisoner's dilemma game, snowdrift game and public good game. As the leading paradigm, the prisoner's dilemma game is widely utilized [5]. Related counter-temptation mechanisms or solutions have been developed to countervail temptation and promote cooperation. Each of them captures certain patterns of human's behavior, such as kin selection [26,27], direct reciprocity [27,28], indirect reciprocity

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$$\begin{matrix} C \\ D \end{matrix} \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}$$

**Fig. 1.** The payoff matrix. Each individual who cooperates gets one if the partner is cooperating. Each one who defects obtains $b$ ($b > 1$) if the partner is cooperating. In other cases, he or she gets noting instead.

[29–32], group selection [33,34], tolerance [35], altruism punishment [36–38], spatially structured populations [8,39–44], social values [13,45], selective investment [3,9,46], social diversity [2,4,7,9,47], volunteerism [37,48–50], aspirations [51,52], multilayer networks [53–55], and complex networks [56,57].

Apart from them, the reputation mechanism is commonly introduced as a necessary countervailing mechanism against temptation [5,6,13,50,58–61]. Some researchers investigated the effects of reputation on the individual partner-switching process, and it indicates that the group's cooperation propensity will increase by almost 100% if individuals are free to alter actions or partners [6]. However, some researchers argue that individuals' reputation cannot be fully identified in reality, due to the cost and error of information dissemination [5,59]. Individuals have limited information and heterogeneous capabilities to identify the neighbors' reputation correctly. Thus, the term of inferring reputation probability is applied to refer to the situation that each agent infers the neighbor* with highest reputation correctly merely with a probability $p$, which is normally distributed. Outcomes show that inferring reputation promotes cooperation more than traditional ways [5,58].

Although we have already known that temptation reduces cooperation substantially [5,6,38,58,59], two issues stay unclear: First, how different levels of $p$ influence the cooperation propensity, i.e. $p$'s effect on cooperation rate; if reputation promotes cooperation, the second issue naturally emerges that when and where reputation effectively countervails temptation, i.e. the condition where reputation countervails temptation. The first question lays the foundation of the second one. This work applies spatial prisoners' dilemma game to investigate and solve these issues.

## 2. Model

Agents commonly play prisoners' dilemma games on a square lattice [5,38] in existing researches. The prisoners' dilemma game is also called as social dilemma game [5,57]. Agents play games with its eight neighbors, and each has two strategy options, which is cooperate and defect that are denoted as $C$ and $D$ respectively. Payoff matrix is shown in Fig. 1, which has only one parameter $b$ that satisfies $b \in (1, 2]$ [5,6]. If one cooperates with a neighbor who cooperates he receives one, if he or she defects with a neighbor who cooperates then the payoff is $b$, and the payoff would be zero otherwise.

According to previous work [5,6], we utilize Eq. (1) to generate reputation for each agent. Initially, each agent is given a reputation of one. Afterwards, reputation is determined by the increment of reputation, or the strategy chosen by each agent $i$ at each time $t$, i.e. $\Delta_i z$. For each agent, $\Delta_i z$ is one if he or she cooperates and zero if he or she defects. The term $Z_i(t-1)$ represents individual's reputation at time $t-1$, and

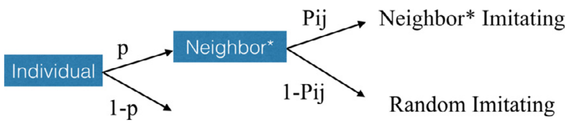


**Fig. 2.** The action rule. Neighbor* refers to the neighbor with the highest reputation. For each agent, there is a referring reputation probability $p$, and each one imitates the action of neighbor* at the probability of $p$. If this agent chose to imitate neighbor*, the probability is $P_{ij}$, i.e. the transition probability. One imitates a random neighbor otherwise.

$Z_i(t)$ is the accumulative current reputation of each one at time $t$.

$$Z_i(t) = Z_i(t-1) + \Delta_i Z \tag{1}$$

Neighbor* denotes the neighbor with highest reputations. Each agent imitates his or her neighbor* with a probability $p$ called inferring reputation probability [5]. In order to investigate effects of $p$, we assume that agents share the same level of $p$ that satisfies $p \in (0, 1]$. Hence, as is indicated in Fig. 2, each agent finds his or her own neighbor* with the probability of $p$, and imitates action of the neighbor* with the transition probability $P_{ij}$. In other cases, agent imitates one neighbor randomly.

The formula of transition probability $P_{ij}$ is shown by Eq. (2), where $s_i$ and $s_j$ represent agent's current action and action of his or her neighbor*. At each time $t$ or interaction, each agent chooses one specific action, either cooperation or defection, and receives a payoff. Terms $u_i$ and $u_j$ are payoffs agent $i$ and $j$, respectively. The focal agent $i$ adopts action of his or her neighbor* $j$ with the transition probability $P_{ij}$. Therefore, the higher the payoff of $j$ is than that of $i$, the more possible it is for the focal agent to imitate action of $j$. Equivalently, the lower the payoff of $j$ is than that of $i$, the less possible for $i$ to imitate action of $j$ [5,6,56].

$$P_{ij} = P_{s_i \to s_j} = \frac{1}{1 + e^{(u_j - u_i)\beta}} \tag{2}$$

In Eq. (2), $\beta$ represents the intensity of selection, which means that $\beta \to 0$ leads to random drift while $\beta \to \infty$ deterministic imitation. Given that $\beta$ is not our focus, we set $\beta \equiv 1$. We set the number of agents 40,000 (200 × 200) and the initial percentage or propensity of cooperation is 50%. The probability $p$ takes on 10 typical values, i.e. $p \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. Likewise, $b$ takes on 5 typical values from interval (1, 2], i.e. $b \in \{1.2, 1.4, 1.6, 1.8, 2.0\}$. For each combination of parameters, the simulation process runs for 800 iterations or periods and the cooperation propensity or percentage $\rho_C$ is recorded at each time or iteration $t$.

## 3. Reputation versus temptation

There exist two mechanisms, temptation and reputation, that jointly determine the cooperation rate $\rho_C$. The temptation mechanism means that how temptation affects cooperation. This mechanism produces a negative effect on cooperation propensity [5,6,38,58]. The reputation mechanism indicates reputation's effect on cooperation, which is the focus of this work. We investigate absolute and relative effects of the two mechanisms on cooperation. The temptation