Chaos, Solitons and Fractals 78 (2015) 285–296

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Investigation of coherent modes in the chaotic time series using empirical mode decomposition and discrete wavelet transform analysis

Pankaj Kumar Shaw*, Debajyoti Saha, Sabuj Ghosh, M.S. Janaki, A.N. Sekar Iyengar

Plasma Physics Division, Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Kolkata 700064, India

A R T I C L E I N F O

Article history:
Received 5 January 2015
Accepted 14 August 2015
Available online 1 September 2015

Keywords:
Coherent mode
Chaotic time series
Empirical mode decomposition
Wavelet analysis
Bicoherency

A B S T R A C T

This paper presents a comparative study on the investigation of coherent modes in chaotic time series data based on two techniques: the empirical mode decomposition and the discrete wavelet transform. We have applied these techniques to the different types of chaotic time series data obtained from a glow discharge plasma. The discrete wavelet transform and empirical mode decomposition analysis of the chaotic time series, combined with some simple statistical estimations like variance and correlation coefficient, helps in identifying the presence of coherent modes. We carried out a bicoherency analysis on the coherent modes extracted using empirical mode decomposition to detect the interactions amongst them. It is quite likely that the interactions between the different plasma modes are responsible for such turbulent nonlinear oscillations.

1. Introduction

It was generally believed that turbulence is a random phenomena, but it has been observed that turbulent flow contains motion with a broadband of scales [1]. There are two different types of scales present in such a flow, one at which most energy resides and another at which energy dissipates. The energy containing scales which exhibit most evident structure are usually referred to as coherent structures [1]. There are several definitions of coherent structures in literature [1–4] but the most relevant one was given by Robinson [4] “Coherent structure is defined as a region of flow over which at least one fundamental flow variable exhibits significant correlation with itself or with another variable over a range of space and/or time that is significantly larger than the smallest local scale of flow”. With the advent of chaos theory, it is now recognized that both chaos and turbulence are closely related [5]. So, we can say a coherent structure (CS) is a mode which coexists in a turbulent or chaotic flow, retaining its form over many characteristic lengths or times and also show a significant correlation with the original flow, and hence can have significant effects on the transport and mixing [6]. Coherent structures are partially deterministic features of a turbulent flow field which have been experimentally observed using schlieren and shadowgraph pictures [7]. Incidentally, these structures which can also exist in a chain of mutually coupled oscillators [8] need not always be periodic or linear, and hence, for their detection, one has to resort to nonlinear techniques.

Turbulent or chaotic time series signals are highly fluctuating, non stationary and intermittent. They also have a broadband feature and may consist of a superposition of localized structures in time. Though there is no clear definition of coherent structure present in a turbulent or chaotic time series data, Farge et al. [9] suggested that coherent structures are “not noise”. Since any experimental time series can consist of noise and coherent feature, the remaining part of time

* Corresponding author. Tel : +91 9804749860.
E-mail address: pankaj.shaw@saha.ac.in, pankaj.shaw05@gmail.com (P.K. Shaw).

http://dx.doi.org/10.1016/j.chaos.2015.08.012

series after denoising the time series can be considered as a coherent structure.

Currently, wavelet transform [9–11] and empirical mode decomposition [12–17] (EMD), time-frequency analysis tools, are commonly used for the processing of non-stationary and nonlinear signals. The wavelet transform decomposes a signal into different frequency bands and at different time points with the help of basis functions. The basis functions have the property of localization in time and frequency. The only difficulty in using the wavelet transforms, is that the choice of the basis functions influences the results [11]. EMD was introduced by Huang et al. [12] for the analysis of non stationary and nonlinear signals and does not require any predefined basis functions as in Fourier or wavelet analysis. Flandrin and Goncalves [13] suggested EMD as a data-driven wavelet like expansion, which also helps to extract the natural oscillation frequency present in the time series. The wavelet transform has been used extensively for denoising and detection of the coherent structures of a time series data [9–11,18]. In the references [10,11], a coherent structure has been identified as the mode with the highest energy concentration but it is possible to have more than one coherent structure. Since EMD has proved to be a data driven wavelet like expansion, it can be used as a tool for the detection of the coherent structures and in this paper we have considered the highest energy mode as well as those with comparable energy to the coherent modes. This allows us to not only identify but also study the interaction between the coherent modes leading to the chaotic behavior.

The detection of coherent structures in a plasma turbulence or chaos is important in view of their role in the transport of momentum and energy [19]. The aim of this paper is to detect the coherent modes in the chaotic time series (CTS) data using EMD and discrete wavelet transform (DWT) analyses [10,11]. The versatility of these techniques has been demonstrated by applying them to experimental time series data obtained from a glow discharge plasma. One can detect the periodic coherent modes using Fourier analysis, but the advantages of EMD and DWT are that as well as periodicity, they can also detect short timescale coherent modes. After identifying the coherent modes by EMD, phase coupling between the different modes has been estimated using bicoherency method which can also give us some information about these modes.

The rest of the paper is organized as follows: Section 2 presents a brief methodology of EMD and DWT. We describe the experimental setup in Section 3 and subsequently show the extraction and analysis of the coherent modes from the experimental chaotic time series data in Section 4. In Section 5 we have discussed the physical contribution of the coherent modes by estimating their frequencies and presented a qualitative comparison between EMD and DWT. Section 6 presents the bicoherency analysis to investigate the interaction amongst the coherent modes. Conclusion and summary of the results are presented in Section 7.

2. Theoretical background

2.1. Empirical mode decomposition

Empirical mode decomposition (EMD) is a method which resolves a signal into its inherent modes, called intrinsic mode functions (IMFs) [12]. In our work, we decompose a signal into its IMFs based on the local time scales i.e. the detection of the local maxima and minima. This method may be applied to both nonlinear as well as non stationary signals. There are two conditions for a signal to be an IMF: (1) envelopes of maxima and minima must have zero mean and (2) the number of extrema and the number of zero crossings differ at most by one. The first condition assures that the IMF is symmetric, and the second condition assures that no riding waves of multiple frequency exist in an IMF. These two conditions ensure that the IMF is monocomponent in frequency [12].

A signal $x(t)$ can be decomposed into its IMFs using the sifting process [12] which involves the following steps:

1. Detect all the local maxima and connect them using cubic spline to form an envelope of maxima, $E_{max}(t)$.
2. Detect all the local minima and connect them using cubic spline to form an envelope of minima, $E_{min}(t)$.
3. Compute the mean $m(t) = \frac{E_{max}(t)+E_{min}(t)}{2}$.
4. Compute the residue $h_{11}(t) = x(t) - m(t)$.
5. Iterate step (1–4) on residue until it has satisfied the stopping criteria (discussed in the next paragraph) and $h_{1n}(t) = C_1(t)$ is designated as the first IMF.
6. Compute the residue $R_1(t) = x(t) - C_1(t)$.
7. Iterate step (1–6) on $R_1(t)$ to compute the second IMF $C_2(t)$ and the residue $R_2(t)$.
8. Iterate step (1–7) on the residue and compute the finite number of modes and a residue $R_k(t) = R_{k-1}(t) - C_k(t)$ which is just the trend.

The above algorithm removes the high frequency oscillation from the data with each repetition resulting in higher IMFs containing a lower frequency oscillation than the earlier one. The sifting procedure mentioned above is continued till a particular stopping criteria is met, ideally when the two conditions for a signal to be an IMF are fulfilled. But imposing a too low threshold for terminating the process may lead to the generation of spurious IMFs. There are many stopping criteria discussed in literature [12,14]. Here, we have adopted a stopping criteria based on two thresholds δ_1 and δ_2, the ratio of mean to the amplitude of the envelopes i.e. on $S(t) = \left|\frac{M(t)}{A(t)}\right|$ which is proposed by Rilling et al. [14]. The two threshold conditions δ_1 and δ_2 are imposed to guarantee globally small fluctuations in the mean while taking large excursions. For a given fraction of time $(1-\alpha)$, $S(t)$ should be less than δ_1 and for the rest of the time $S(t)$ should be less than δ_2. In our later analysis, δ_1, δ_2 and α are set at values of 0.05, 0.5 and 0.05, respectively.

The correlation coefficient (CC) of an IMF, gives an idea about its contribution to the original signal, and is calculated using the following relation [15]

$$CC = \frac{\int IMF(t) \cdot X(t)dt}{\sqrt{\int IMF(t) \cdot IMF(t)dt}\sqrt{\int X(t) \cdot X(t)dt}}, \quad (1)$$

where $X(t)$ is a signal. The value of CC is normalized to lie between 0 and 1. Only those modes which have a CC > 0.1(10%) are taken into consideration for the physically significant modes (relevant) and the rest of the modes are considered as redundant.