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Stability analysis of fractional-order complex-valued neural networks with time delays ☆

R. Rakkiyappan [a], G. Velmurugan [a], Jinde Cao [b,c,*]

[a] Department of Mathematics, Bharathiar University, Coimbatore 641 046, Tamilnadu, India
[b] Department of Mathematics, and Research Center for Complex Systems and Network Sciences, Southeast University, Nanjing 210096 Jiangsu, China
[c] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

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ABSTRACT

In this paper, we consider the problem of stability analysis of fractional-order complex-valued Hopfield neural networks with time delays, which have been extensively investigated. Moreover, the fractional-order complex-valued Hopfield neural networks with hub structure and time delays are studied, and two types of fractional-order complex-valued Hopfield neural networks with different ring structures and time delays are also discussed. Some sufficient conditions are derived by using stability theorem of linear fractional-order systems to ensure the stability of the considered systems with hub structure. In addition, some sufficient conditions for the stability of considered systems with different ring structures are also obtained. Finally, three numerical examples are given to illustrate the effectiveness of our theoretical results.

1. Introduction

Fractional calculus was began in the late 17th century. Leibniz, Euler, Fourier and others attempted to extend the concept of derivatives from positive integer index to fractional index. In 1832–1837, Joseph Liouville published the first large-scale connected theory of fractional calculus, which he applied to solve some integral equations earlier than Abel, who is usually credited with being the first. Liouville's fractional calculus was extended by Riemann, and the standard modern fractional calculus is based on the Liouville–Riemann definition of a derivative of arbitrary index (Lützen [30], Chapter 8). Near the end of the 19th century, the electrical engineer Oliver Heaviside applied fractional derivatives, but otherwise they attracted little attention until 1980 [18]. Since then, fractional derivatives have been applied extensively by many researchers, scientists and engineers due to the development of widespread applications in various disciplines, namely, Physics, Control Engineering and Signal Processing, see [18,19,28,29,35,37] and references therein. Moreover, many of the real-world problems are modelled by fractional-order dynamical systems rather than integer-order ones, since the fractional-order dynamical systems provide more accurate results than their integer-order counterparts. However, the main reason for the wide applications of fractional calculus is that fractional

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* Corresponding author at: Department of Mathematics, and Research Center for Complex Systems and Network Sciences, Southeast University, Nanjing 210096, Jiangsu, China. Tel.: +86- 25- 83792315; fax: +86- 25- 83792316.

E-mail address: jdcao@seu.edu.cn (J. Cao).

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derivatives provide an efficient and an excellent tool for the description of memory and genetic properties of diverse materials and operations compared to integral-order derivatives. It is well known that, there are two advantages in fractional order models: one is more degrees of freedom in the models and the another is "memory" in the models. Thus, it is necessary and interesting to study the fractional-order dynamical systems, both in theory and in applications, and many researchers have extensively studied the problem of fractional-order dynamical systems and some important and interesting results were proposed in the existing literature: see [12,17,31–33,38,42] and references therein.

In the past few decades, neural networks have received increasing interest and more attention from many scientists and engineers, due to the wide range of applications in various fields of science and engineering. There are several types of neural networks which have been investigated intensively, and some good results have been reported on various topics namely, artificial neural networks, bidirectional associative memory (BAM) neural networks, Hopfield neural networks, cellular neural networks, Switched neural networks and Cohen–Grossberg neural networks. In particular, the Hopfield neural network model was proposed by Hopfield in 1984 [21], and it has been used successfully in various fields such as pattern recognition, knowledge acquisition, associative memory and combinatorial optimization: see [15,39,50] and references therein. The authors have been studied the problem of synchronization of switched neural networks with interval parameters uncertainty (see, [5]), based on the theories of the switched systems and drive-response technique and also discussed the applications to chaotic neural networks. Those practical applications depended heavily on the dynamical behaviors, such as Lyapunov stability and asymptotic stability of neural models. Therefore, the stability analysis of Hopfield neural networks has attracted much attention from many authors, and some excellent results have been published: see [6,14,22,51].

In recent years, fractional calculus has been integrated into artificial neural networks, mainly because it can represent infinite memory, and the fractional-order expression of artificial neural network model has also been proposed in research results about biological neurons. Recently, there has been an increasing interest in the investigation of fractional-order neural networks, and some important and interesting results were obtained in both theory and applications: see [1,2,4,7,25,26,34,40,41,46,47,49,52,53] and references therein.

In [34], the authors pointed out that fractional differentiation provides neurons with a fundamental and general computation ability that can contribute to efficient information processing, stimulus anticipation and frequency-independent phase shifts of oscillatory neuronal firing. Some sufficient conditions have been established for the existence, uniqueness and uniform stability of a class of fractional-order neural networks with time delay, in [7]. In [40], the authors widely investigated the dynamical properties of fractional-order neural networks. The problem of projective synchronization of fractional-order memristor-based neural networks have been considered and studied in [2]. In [1], the authors using the theory of fractional calculus and generalized Gronwall–Bellman inequality approach, some finite-time stability criterion have been established for Caputo fractional neural networks with distributed delay. Furthermore, a generalized capacitor, called "fractance", is often thought to be the main operator in fractional calculus. In [46], the authors pointed out that the common capacitance from the continuous-time integer-order Hopfield neural network can be replaced by the fractance, giving birth to the so-called fractional-order Hopfield neural network model. Several topics related to the dynamics of fractional-order Hopfield neural networks with ring or hub structures were investigated in [26,27], such as stability and multi-stability, bifurcations and chaos. The authors have extensively investigated the problem of stability analysis of fractional-order Hopfield neural networks with time delays, and some sufficient conditions for stability of the proposed networks with hub structure and two types of ring structure have been derived in [46,47]. The robust stability of fractional-order Hopfield neural networks with parameter uncertainties have been potentially investigated in [52]. Some sufficient conditions have been proposed to ensure the existence, uniqueness, and global Mittag–Leffler stability of the equilibrium point by employing suitable Lyapunov functionals. Those authors proposed fractional-order Hopfield neural networks, and investigated its stability through energy-like function analysis, in [4].

On the other hand, most of the practical applications of neural networks are developed in the field of complex domain; that is, its states, connection weights and activation functions are expressed in terms of complex-valued data, and it processes information on the complex plane. Therefore, it is known as complex-valued neural networks (CVNNs) which explore new capabilities and higher performance, making it possible to resolve problems which cannot be solved with their real-valued counterparts. Recently, there has been increasing interest on studying the dynamic behaviors of CVNNs due to their widespread applications in filtering, imaging, optoelectroincs, speech synthesis, computer vision, see [20,36,43,45] and references therein. Moreover, the activation function plays an important role in the dynamic analysis of CVNNs. In CVNNs, there are two types of activation function, one is real/imaginary-type activation function and the other is amplitude/phase-type activation function. Besides, CVNNs have different and more complicated properties than real-valued ones. Likewise, in the electronic implementation of parallel neural networks, time delay is usually unavoidable due to the finite switching speed of amplifiers, and it may cause oscillations or instability of dynamic systems. Thus, it is important to study the dynamic behaviors of CVNNs with time-varying delays. Some authors have focused their attention to analyze the dynamic behaviors of CVNNs with time-varying delays and have proposed some excellent results in the literature, see [8–10,23,24,48,54,55] and references therein. In [23], authors have studied the problem of global stability of CVNNs with time delays, and several sufficient conditions have been derived to ascertain the existence of unique equilibrium, global asymptotic stability, and global exponential stability of delayed CVNNs with both classes of complex-valued activation functions. The authors in [55] have discussed a class of discrete-time recurrent neural networks with complex-valued linear threshold neurons, and some sufficient conditions have been obtained for boundedness, global attractivity and complete stability of proposed networks. By using appropriate Lyapunov–Krasovskii functional and employing the free-weighting matrix method, some sufficient conditions have been given to ensure the global μ-stability of CVNNs with unbounded time-varying delays, in [9]. In [48], the authors have widely investigated the dynamic behaviors of a class of